On the Relation Between Base-rate and Cost-Benefit Learning in Simulated Medical Diagnosis

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Abstract

Observers completed a series of simulated medical diagnosis tasks that differed in category discriminability and base-rate/cost-benefit ratio. Point, accuracy, and decision criterion estimates were closer to optimal (a) for category $d' = 2.2$ than for category $d' = 1.0$ or $3.2$, (b) when base-rates, as opposed to cost-benefits were manipulated, and (c) when the cost of an incorrect response resulted in no point loss (non-negative cost) as opposed to a point loss (negative cost). These results support the “flat-maxima” (von Winterfeldt & Edwards, 1982) and COMpetitiOn Between Reward and Accuracy (COBRA; Maddox & Bohil, 1998a) hypotheses. A hybrid model that instantiated simultaneously both hypotheses was applied to the data. The model parameters indicated that (a) the reward-maximizing decision criterion quickly approached the optimal criterion, (b) the importance placed on accuracy maximization early in learning was larger when the cost of an incorrect response was negative as opposed to non-negative, and (c) by the end of training the importance placed on accuracy was equal for negative and non-negative costs.

Introduction

Each day we are faced with situations in which we must decide a course of action based on uncertain information. For example, we might decide to “wear” or “not wear” a coat based on a quick look outside at the level of overcast, to “hire” or “not hire” an applicant based on their GPA, or to diagnose “a heart attack” or “indigestion” based on the degree of chest pain. These are all categorization problems because there are many possible levels of overcast, GPA, and chest pain, but only two potential decisions, “wear” or “not wear” a coat, “hire” or “not hire” an applicant, and “diagnose” or “do not diagnose” a heart attack. In each case, we can categorize correctly or incorrectly. For example, a patient who just experienced a heart attack can be correctly diagnosed, or can be incorrectly diagnosed. Categorization problems of this sort differ along at least three factors: the discriminability of the categories, the prior probabilities (or base-rates) of the categories, and the costs and benefits associated with each categorization decision.

Different sources of information yield different levels of category discriminability. For example, a patient’s degree of chest pain might be a good discriminator between heart attack and no heart attack, but might be a poor discriminator between cancer and no cancer. Throughout this article we will use the term category discriminability to refer to the standardized distance between category means; also called category $d'$ (Green & Swets, 1966; Macmillan & Creelman, 1991). The category base-rates might also differ. For example, if most of our patients have just experienced a heart attack (perhaps because of some stressful event like an earthquake) they will likely evidence chest pain, and will likely be diagnosed correctly. Finally, the costs and benefits associated with each categorization decision might differ. We generally benefit when we make the correct decision. However, if we are dealing with a life or death situation, then the benefit of a correct heart attack diagnosis might be greater than the benefit of a correct indigestion diagnosis. Similarly, there is often a cost associated with an incorrect decision; a more severe cost might be incurred if a heart attack patient is misdiagnosed as having indigestion, than if an indigestion sufferer is misdiagnosed as having a heart attack.

When presented with a categorization problem of this sort, the optimal classifier (a hypothetical device) uses a
decision strategy that maximizes long-run expected reward. When a single dimension of information is used to solve the categorization problem, the optimal classifier sets a decision criterion along the dimension and gives one categorization response for dimensional values below the criterion and gives the other categorization response for dimensional values above the criterion. The location of the optimal decision criterion is determined by the category base-rates and cost-benefits. The stimulus dimension value used to generate a categorization response is influenced by the category discriminability. Thus, the optimal classifier is sensitive to category discriminability, base-rate and cost-benefit information, and uses this information to set a decision criterion that maximizes long-run expected reward.

This article reports the results of a categorization experiment, a simulated medical diagnosis task, in which several category discriminability (d’), base-rate and cost-benefit conditions were combined factorially. Every observer completed several blocks of trials in all of the resulting categorization conditions. We investigated how the observer’s decision criterion varied as a function of category d’, base-rate, and cost-benefit condition, and how the observer’s decision criterion changed across blocks of trials within a condition.

Our data analytic approach was to develop and test a series of decision bound models, each of which was applied simultaneously to the data from all experimental conditions separately for each observer and block of trials (Bohil & Maddox, in press; Maddox & Ashby, 1993; Maddox & Bohil, 1998a, 2000). Decision bound models assume that human observers try to apply a decision criterion, like the optimal classifier, but have less success because of inherent sub-optimalities (described later) in the human perceptual and cognitive systems (e.g., Maddox & Ashby, 1993). Each model instantiated a different hypothesis regarding the effects of category discriminability, base-rate, or cost-benefit manipulations on the decision criterion. Two specific hypotheses will be outlined shortly, and the details of these models will be left for the Results and Theoretical Analyses section.

The next (second) section outlines the effects of category discriminability, base-rate, and cost-benefit manipulations on the performance of the optimal classifier, and reviews briefly the relevant categorization and decision-making literature. The third section outlines the experiment and the methods, and the fourth section is devoted to the results and theoretical analyses. Finally, we conclude with some general comments.

Optimal Classifier

The optimal classifier is a hypothetical device that maximizes long-run expected reward. Consider the situation facing a medical doctor who must classify a patient into one of two disease categories, A or B. Suppose the patient is given medical test X, which is diagnostic of the two diseases. In addition, suppose that the outcomes of test X for diseases A and B are normally distributed with means $\mu_A$ and $\mu_B$, and standard deviation $\sigma_A$ and $\sigma_B$, as depicted in Figure 1a. The optimal classifier records perfectly the test result, denoted x. The optimal classifier has perfect knowledge of the distribution of test results for each disease category (i.e., the form and parameters of the distribution). This information is used to construct the optimal decision function, which is computed from the likelihood ratio of the two category distributions,

$$l(x) = \frac{f(x|B)}{f(x|A)}$$

where $f(x|i)$ denotes the likelihood of test result x given disease category i. If test result x is more likely to result from disease B than disease A, then the likelihood ratio will be greater than one. If test result x is more likely to result from disease A than disease B, then the likelihood ratio will be less than one. The likelihood ratio for any given value x will be affected by the category discriminability, d’. Figure 1 depicts categorization problems for three different levels of category discriminability, d’ = 1, 2.2, and 3.2.

The optimal classifier has perfect knowledge of the category base-rates, the costs associated with incorrect diagnoses, and the benefits associated with correct diagnoses. This information is used to construct the optimal decision criterion

$$\beta_o = \frac{[P(A)(V_{aA} - V_{bA})]/[P(B)(V_{bB} - V_{aB})]}{[P(B)(V_{bB} - V_{aB})]/[P(A)(V_{aA} - V_{bA})]}$$

where P(A) and P(B) are the base-rate probabilities for categories A and B, $V_{aA}$ and $V_{bB}$ denote the benefits associated with correct diagnoses, and $V_{bA}$ and $V_{aB}$ denote the costs associated with incorrect diagnoses (Lower-case subscripts denote responses, and upper-case subscripts denote categories). The costs and benefits make up the elements of the payoff matrix. The optimal classifier (e.g., Green & Swets, 1966) uses l(x) and $\beta_o$ to construct the optimal decision rule:

If $l(x) > \beta_o$, then respond “B”, otherwise respond “A”.

Notice that when the base-rates, and cost-benefit differences are equal across categories [i.e., when P(A) = P(B), and $(V_{aA} - V_{bA}) = (V_{bB} - V_{aB})$, then $\beta_o = 1$, and the optimal classifier assigns the stimulus to the category with the highest likelihood. This situation is depicted in Figure 1 by the $\beta_o = 1.0$ decision criterion. Notice also that base-rate
and cost-benefit manipulations have the same effect on the optimal decision criterion. For example, suppose disease A is three times as common as disease B, and the payoff matrix is symmetric [i.e., if \( P(A) > 3P(B) \) and \( (V_{aA} - V_{bA}) = (V_{bB} - V_{aB}) \)], or suppose the cost-benefit difference for disease A is three times larger than for disease B and the base-rates are equal [i.e., when \( (V_{aA} - V_{bA}) > 3(V_{bB} - V_{aB}) \) and \( P(A) = P(B) \)], then \( \beta_o = 3.0 \) (see Figure 1). In this case, the optimal classifier will generate a disease A diagnosis unless the likelihood of disease B is at least three times larger than the likelihood of disease A. Base-rate and cost-benefit differences are ubiquitous in the real-world, especially in the field of medicine where some diseases are more common than others, and some diseases are life-threatening, whereas others are not.

Before continuing, one important issue regarding the optimal classifier needs to be discussed. The optimal decision criterion is constructed from the “objective” or “true” category information. Some decision-making research suggests that people do not use the objective costs and benefits, but rather base their decisions on subjective costs and benefits that are directly related to the objective values (e.g., Kahneman & Tversky, 1979; Stevenson, Busemeyer, & Naylor, 1991; Tversky & Kahneman, 1974, 1980, 1992; Yates, 1990). Within the framework of decision theory, each of our \( V_{ij} \) terms should be converted into a subjective utility denoted \( u(V_{ij}) \), where \( u \) describes the functional relationship between the subjective and objective values. In the case of points converted to money, it is reasonable to assume that increasing value is associated with increasing utility. Many functions of this sort are possible, but decision theorists have focused on concave, convex, and linear relationships. Although a detailed examination of these functional relations and their implications for decision-making and categorization problems is beyond the scope of this article, the linear function is most directly relevant to the current study. As long as the utility function, \( u \), is linear, then the optimal decision criterion is equivalent to the expected utility criterion. This is a strong assumption, but one that should hold (approximately) for the small range of monetary values in the present study. Support for the linearity assumption is offered in the Results section. In addition, an alternative nonlinear formulation is outlined in the General Discussion.

Category Discriminability and the “Flat-Maxima” Hypothesis of Decision Criterion Learning

Some categorization and decision-making studies have examined systematically the effects of category discriminability on performance (e.g., Bohil & Maddox, in press; Kubovy & Healy, 1980; Lee & Zentall, 1966; Wallsten, Bender, & Li, 1999; see also Wallsten & Gonzalez-Vallejo, 1994). Most relevant to the current study, Bohil and Maddox (in press) had each observer complete categorization problems in which there was a 3:1 base-rate ratio, a 3:1 benefit ratio (costs of incorrect responses were always set to zero), and various base-rate/benefit mixtures at two category \( d' \) levels, \( d' = 1.0 \) and \( d' = 2.2 \). These category \( d' \) levels were chosen to test a specific hypothesis regarding the effects of category discriminability on decision criterion placement called the “flat-maxima” hypothesis. As suggested by many researchers, suppose that the observer adjusts their decision criterion based (at least in part) on the change in the rate of reward, with larger changes in rate being associated with faster, more nearly optimal decision criterion learning (e.g., Busemeyer & Myung, 1992; Dusoir, 1980; Erev, Gopher, Itkin, & Greenshpan, 1995; Erev, 1998; Kubovy & Healy, 1977; Roth & Erev, 1995; Thomas & Legge, 1970). To formalize this hypothesis one can construct the Objective Reward Function (ORF) that plots the objective expected reward as a function of the decision criterion, \( k \). Figure 2a shows the ORF for category \( d' = 1.0 \) and category \( d' = 2.2 \) for a 3:1 base-rate or 3:1 cost-benefit ratio (for now ignore the function labeled category \( d' = 3.2 \)). Specifically, Figure 2a plots expected reward as a function of the deviation between the observer’s decision criterion (\( \beta \)) and the optimal decision criterion (\( \beta_o \)) standardized by category \( d' \). This \( k - k_o \) measure = \( \ln(\beta)/d' - \ln(\beta_o)/d' = \ln(\beta/\beta_o)/d' \) is the ratio of the actual and optimal decision criterion standardized by category \( d' \). The derivative of an ORF at a specific \( k - k_o \) value determines the change in the rate of the expected reward for that ORF at that \( k - k_o \) value; the larger the change in the rate, the “steeper” the ORF at that point. Figure 2b plots the relationship between the steep-ness for each ORF (i.e., the derivatives for each ORF) and \( k - k_o \). The horizontal line on Figure 2b denotes a fixed steep-ness value, and the vertical lines denote the associated \( k - k_o \) values for each category \( d' \). Several comments are in order. First, notice that for a fixed non-zero steep-ness the decision criterion, \( k \) that results differs across category \( d' \) conditions—that is, the \( k \) value associated with a fixed steepness differs across category \( d' \). Second, notice that the decision criterion, \( k \), is closer to the optimal value, \( k_o \), for category \( d' = 2.2 \), than for category \( d' = 1.0 \) for all non-zero steepness values. Finally, if the observer adjusts their decision criterion based on the change in the rate of reward (or steep-ness of the ORF), as described above, then steeper
ORF's should be associated with more nearly optimal decision criterion. Similarly, flatter ORF's should be associated with more sub-optimal decision criterion. vonWinterfeld and Edwards (1982; see also Kubovy & Healy, 1980) referred to this as the “flat-maxima” hypothesis. Bohil and Maddox (in press) found support for the “flat-maxima” hypothesis in their data.

This “flat-maxima” hypothesis holds for most values of k, and for any linear utility function (vonWinterfeld & Edwards, 1982). It also holds for most nonlinear utility functions as long as the utility is not a function of category d’.

Although a nonlinear utility function could change the steepness of the ORF, it generally would not change the rank ordering of ORF steepness for the three category d’ values unless the utility was a function of category d’.

Although the “flat-maxima” hypothesis predicts better decision criterion learning in the category d’ = 2.2 condition over the category d’ = 1.0 condition, a closer examination of Figures 1 and 2 suggest at least two alternative explanations. First, notice that the maximum attainable accuracy and maximum attainable reward is higher when category d’ = 2.2 than when category d’ = 1.0. The fact that accuracy is higher when category d’ = 2.2 can be seen by noting that the overlap of one category into the response region for the other category is smaller in Figure 1b than in Figure 1a (see also Table 2). The fact that reward increases with category d’ can be seen in Figure 2a by noting that the peak of the ORF is higher for category d’ = 2.2 than for category d’ = 1.0 (see also Table 2). Thus, an alternative explanation for the Bohil and Maddox (in press) finding is that decision criterion learning is better when higher levels of performance (i.e., accuracy and reward) can be achieved. Second, notice that the stimulus value, x, associated with the k = 3 decision criterion is closer to the stimulus value, x, associated with the equal likelihood criterion k = 1 for category d’ = 2.2 than for category d’ = 1.0 (compare Figures 1a and 1b). Thus, a second alternative explanation is that decision criterion learning is better when the stimulus values associated with k = 1 and k = 3 are closer together. Unfortunately, in the Bohil and Maddox (in press) study these latter two hypotheses, as well as the theoretically motivated “flat-maxima” hypothesis, make identical predictions and thus a critical test is not possible.

One focus of the current study was to provide a critical test of these three hypotheses by examining decision criterion learning for category d’ = 1.0, 2.2, and 3.2 (see Figure 1c). The third category d’ level was chosen so that the “flat-maxima” hypothesis makes a qualitatively different prediction from the prediction for the latter two hypotheses. The hypothesis that an increase in maximum attainable accuracy and reward leads to better decision criterion learning predicts a monotonic increase in decision criterion learning with increases in category d’. Similarly, the hypothesis that decision criterion learning will be better when the stimulus values associated with k = 1 and k = 3 are closer together in the stimulus space makes the same prediction. In other words, both hypotheses predict that the k-k, values will be smallest for d’ = 3.2, intermediate for d’ = 2.2, and largest for d’ = 1.0. Only the “flat-maxima” hypothesis makes a different prediction. From Figure 2a notice that the ORF is steepest for category d’ = 2.2, is flattest for category d’ = 1.0, and is intermediate for category d’ = 3.2. Thus, from Figure 2b, notice that the “flat-maxima” hypothesis predicts that performance should be closest to optimal when category d’ = 2.2, should be farthest from optimal when category d’ = 1.0, and should be intermediate when category d’ = 3.2. In other words, the “flat-maxima” hypothesis predicts that the k-k, values will be smallest for d’ = 2.2, intermediate for d’ = 3.2, and largest for d’ = 1.0.

Unequal Base-rates, Asymmetric Payoff Matrices, and the COMPetition between Reward and Accuracy Hypothesis (COBRA; Maddox & Bohil, 1998a) of Decision Criterion Learning

The optimal decision criterion is affected in the same manner by unequal base-rate ratios and asymmetric payoff matrices. For example, the optimal decision criterion when P(A)=3P(B) and (V_a/V_b)=(V_b/V_a) hold is equivalent to the optimal decision criterion when P(A)=P(B) and (V_aV_b)=(3V_bV_a) hold. Even so, a ubiquitous finding in the categorization literature is that the observer’s decision criterion is consistently closer to the optimal decision criterion when base-rates are unequal, than when payoff matrices are asymmetric (Bohil & Maddox, in press; Healy & Kubovy, 1981; Kubovy & Healy, 1980; Maddox, 1995; Maddox & Bohil, 1998a, 1998b; Weber, Bockenholt, Hilton, & Wallace, 1993). Maddox and Bohil (1998a) offered a COMPetition Between Reward and Accuracy maximization (COBRA) hypothesis that predicts this pattern of results. The idea is that observers attempt to maximize long-run expected reward as instructed but also place some importance on the accuracy of their responses. When base-rates are manipulated and the payoff matrix is symmetric, both goals can be achieved simultaneously because the decision criterion that maximizes reward also maximizes accuracy. For example, consider the two-category problem depicted in Figure 3a. In this case, base-rates are manipulated and the payoff matrix is symmetric. The decision criteria k_a and k_b denote the decision criteria that maximize expected reward, and expected accuracy, respectively. Notice that k_a = k_b and thus expected reward and expected accuracy can be maximized simultaneously. On the other hand, when the payoff matrix
is asymmetric and base-rates are equal, both goals cannot be achieved simultaneously because the decision criterion that maximizes expected accuracy is different from the decision criterion that maximizes expected reward. For example, the two-category problem depicted in Figure 3b shows a case in which base-rates are equal, but the payoff matrix is asymmetric. Notice that the decision criterion that maximizes expected reward \( k_r \) is different from the decision criterion that maximizes expected accuracy \( k_a \). Thus, an observer who places importance (or weight) on both goals will show less optimal performance in the asymmetric payoff condition than in the unequal base-rate condition. To instantiate this hypothesis we assume a simple weighting function, \( k = wk_a + (1-w) k_r \), where \( w \) \((0 \leq w \leq 1)\) denotes the weight placed on expected accuracy maximization. This weighting function results in a single decision criterion that is intermediate between that for accuracy maximization and that for reward maximization. In Figure 3b, \( k_1 \) denotes a case in which \( w < .5 \), whereas \( k_2 \) denotes a case in which \( w > .5 \).

Other weighting schemes are possible. For example, instead of generating an intermediate decision criterion, it is possible that the two decision criteria compete on each trial for the opportunity to generate the categorization response (for related proposals see Ashby, Alfonso-Reese, Turken, & Waldron, 1998; Maddox & Estes, 1996). Another possibility is that the utility of the cost associated with an error is larger than the true cost, effectively decreasing the magnitude of the optimal decision criterion. A rigorous comparison of these alternatives is beyond the scope of this article, although we revisit the latter possibility in the General Discussion.

Recently, Maddox and Bohil (2000; see also Busemeyer & Myung, 1992; Barkan, Zohar, & Erev, 1998 and from the decision-making literature see Busemeyer, 1985; Busemeyer & Rapoport, 1988; Wallsten, et al, 1999) examined the effects of negative vs. non-negative costs in asymmetric payoff conditions on decision criterion learning in a categorization problem with category \( d' = 2.2 \). A negative cost results when the observer loses points for an incorrect response. A non-negative cost results when the observer loses no points, or possibly gains a small amount of points for an incorrect response. They found more nearly optimal decision criterion when the costs were non-negative, as opposed to negative. This result is in line with some decision-making research that suggests that observers often place more weight on costs than on benefits (e.g., Busemeyer, 1982; Busemeyer & Rapoport, 1988; Kahneman & Tversky, 1979; see also Higgins, 1987), but is at odds with some probability learning research which suggests that negative costs lead to faster learning (e.g., Siegel & Goldstein, 1959; Bereby-Meyer & Erev, 1998). A comparison of categorization and probability learning will be reserved for the General Discussion, but for now it seems reasonable to attempt first to replicate the Maddox and Bohil (2000) findings, and to extend this investigation to other category \( d' \) values.

To summarize, the COBRA hypothesis and the Maddox and Bohil (2000) results suggest that the decision criterion should be closest to optimal when base-rates are unequal, farthest from optimal when payoff matrices are asymmetric and costs are negative, and intermediate when payoff matrices are asymmetric and costs are non-negative.

Experiment

The over-riding goal of this research was to provide a rigorous empirical examination of the effects of category discriminability, base-rate/cost-benefit, and negative/non-negative cost manipulations on the time-course of decision criterion learning in a simulated medical diagnosis task. This research informs the categorization community but also helps bridge the gap between the related, but too often separate, fields of categorization and decision-making. One 3:1 base-rate [i.e., \( P(A) = .75 \), \( P(B) = .25 \)], and four 3:1 cost-benefit conditions were combined factorially with three category discriminabilities, for a total of 15 experimental conditions. The base-rates and payoff matrix entries for all five experimental conditions are displayed in Table 1. The 3:1 base-rate and non-negative cost(A) conditions are the same as those used in Maddox and Bohil (1998a; Bohil & Maddox, in press). A second non-negative cost(B) condition was also included. The two negative cost conditions [negative cost(A) and negative cost(B)] were taken from Maddox and Bohil (2000). To anticipate, few performance differences were observed between the two negative cost conditions, and between the two non-negative cost conditions. Thus, the majority of the analyses focus solely on the negative/non-negative distinction.

A within-observer design was utilized. The five experimental conditions for a given level of category discriminability were run during a one-week period. Prior to the start of the first experimental session, each observer completed a full session in a baseline condition in which no base-rate nor cost-benefit manipulation was present. The baseline condition was included to ensure that observers had accurate knowledge of the category distributions prior to any base-rate or cost-benefit manipulations.

The data analyses focus on the performance of a series of quantitative models applied simultaneously to the data.
from each of the 15 experimental conditions separately for each observer and block of trials. Each model instantiated a
different set of hypotheses about the effects of the category discriminability, base-rate/cost-benefit, and negative/non-
negative cost manipulations on decision criterion placement. In particular, we developed models that instantiated strong
versions of the “flat-maxima” and COBRA hypotheses, as well as a hybrid that incorporated the assumptions of both
hypotheses within the framework of a single model (detailed below). Learning effects were examined by comparing
performance of the various models, and changes in the model parameter values across blocks of trials. All analyses
were performed at the individual-observer level, because of concerns with modeling aggregate data (e.g., Ashby,

To anticipate, we found that the observer’s decision criterion placement was determined by three factors: (a) the
location of the optimal decision criterion, (b) the steepness of the ORF, and (c) a tradeoff between responding that
maximized long-run accuracy and responding that maximized long-run reward.

Method

Observers. Six observers were solicited from the University of Texas community. All observers claimed to have 20/20
vision or vision corrected to 20/20. Each observer completed 18 approximately 30-minute sessions. The task on each
trial of each session was identical. The observer was presented with a stimulus, was asked to determine whether it was
a member of category “A” or category “B”, and was given corrective feedback following each response. Observers
were paid based on their day-to-day performance in the task.

Stimuli and Stimulus Generation. The stimulus was a filled white rectangular bar that varied in length from trial to trial
(40 pixels wide) set flush upon a stationary base (60 pixels wide). A sample stimulus is displayed in Figure 4. There
were two categories of bar heights, A and B, each defined by a specific univariate normal distribution (Ashby & Gott,
1988). The separation between the Category A and B means were 21, 45, and 67 pixels for category d’ = 1.0, 2.2, and
3.2, respectively. The standard deviation for Category A and B was 21 pixels for all three category d’ levels.

For the category d’ = 1.0 condition, two sets of 60 stimuli were generated. One set was used in all cases for
which the base-rates were equal (baseline and cost-benefit manipulation conditions), and the other was used in the 3:1
base-rate condition. Each set was generated by taking numerous random samples of size 60 from the population and by
selecting the sample that best matched the population objective reward function. Stimuli for the category d’ = 2.2 and
category d’ = 3.2 conditions were generated from the category d’ = 1.0 samples by applying the appropriate
transformation.

Three measures were taken to discourage information transfer across category d’ conditions. First, the bar
length associated with the optimal decision criterion in the baseline condition was varied across levels of category d’.
Second, whether the bars varied vertically in height or horizontally in length differed across levels of category d’.
Finally, across category d’ conditions, different category labels were used (e.g., “burlosis” and “namitis” in one
category d’ condition, and “catarria” and “phlebarrah” in another category d’ condition).

Each session in the experiment consisted of 6 60-trial training blocks, followed by a 120 trial transfer block.
During training, corrective feedback was provided on each trial (see details below). During transfer, no feedback was
provided. The same 60 stimuli were presented once in each training block and twice in the transfer block, but the
presentation order was randomized across blocks.

In the baseline and 3:1 base-rate condition the cost of an incorrect response was zero (i.e., V_{ab} = V_{ba} = 0), and
the benefit of a correct response was 2 points (i.e., V_{aa} = V_{bb} = 2). The costs and benefits associated with A and B
responses in each of the four cost-benefit experimental conditions are displayed in Table 1. Table 2 displays the point
totals, accuracy rates, and \( \beta \) values for the optimal classifier separately for each condition. In the baseline condition, \( \beta_o = 1 \), and in all experimental conditions, \( \beta_o = 3 \).

Procedure. Observers were told that perfect performance was impossible. However, an optimal level of performance
was specified as a goal (in the form of desired point totals). Observers were told that they were participating in a
hypothetical medical diagnosis task, and the length of the bar represented the results of a particular medical test. The
test was designed to distinguish between two diseases, such as “burlosis” and “namitis”, hereafter referred to as simply A and B. Observers were informed that they would receive the medical test result for a new patient on each trial, and that their goal was to maximize points in each session. They were informed that these point totals would be converted into monetary values that they would receive at the end of the experiment. Observers were instructed to maximize points, and not worry about speed of responding. A typical trial proceeded as follows. A stimulus was presented on the screen, and remained until a response was made (see Figure 4). The observer’s task was to classify the presented stimulus as a member of Category A or Category B by pressing the appropriate button. During the training phase, the observer’s response was followed by 750ms of feedback. Three lines of feedback were presented. The top line indicated the number of points the observer earned for the response. The next line indicated the potential point earnings for a correct response on each trial (i.e., if an observer responded incorrectly, this line indicated the amount that could have been earned had they chosen the correct response). The third line indicated the amount of money that the observer had accumulated up to that point in the session. The feedback was followed by a 125ms inter-trial interval in which the screen was blank. During the transfer phase, the observer’s response was followed by an 875ms inter-trial interval in which the screen was blank. Observers were given a break after each block of trials. At each break, the observer’s accumulated point total was displayed.

The order of the three category d’ conditions was counterbalanced in the following manner. During the first session, each observer completed 6 60-trial training blocks followed by a final 120-trial transfer block in the baseline condition. The baseline condition was completed first to ensure that each observer had accurate knowledge of the category structures before exposure to the base-rate or cost-benefit manipulations. The five experimental conditions were completed, one session per day. During the second and third sessions, the 3:1 base-rate and non-negative cost(A) conditions were completed in a counterbalanced order. During the fourth, fifth, and sixth sessions the two negative cost(A and B) and non-negative cost (B) conditions were completed, again in a counterbalanced order. Before each experimental session, the observer completed a minimum of 60 baseline trials. If the observer reached an accuracy-based performance criterion (no more than 2% below optimal), then they were allowed to begin the experimental condition. If the observer did not reach criterion, they continued in the baseline condition until criterion was reached. Once they reached criterion, they were allowed to begin the experimental condition. Including these baseline trials prior to each experimental condition ensured that each observer had accurate knowledge of the category structures before exposure to the base-rate or cost-benefit manipulation, and minimized the possibility of within-observer carry-over effects from one experimental condition to the next. The category label associated with the high base-rate or high payoff category was randomized across sessions. To simplify the analyses, all data were reorganized so that Category A was the high base-rate or cost-benefit category.

Results and Theoretical Analysis

All analyses were performed on the six blocks of training data, and on the transfer data. We begin with a discussion of baseline condition performance and provide evidence that observers were able to learn the category distributions and use the optimal decision criterion. We then turn to a discussion of the experimental conditions.

To determine whether baseline condition performance was nearly optimal, and to summarize the main trends in the experimental condition data we computed three performance measures. First, we computed the deviation between observed and optimal accuracy standardized by the difference between optimal and zero percent accuracy. Specifically,

\[ \text{accuracy deviation} = \frac{100(\text{observed accuracy} - \text{optimal accuracy})}{\text{optimal accuracy} - 0}. \]

Second, we computed a similar measure for points:

\[ \text{point deviation} = \frac{100(\text{observed points} - \text{optimal points})}{\text{optimal points} - \text{points for 0\% correct}}. \]

Notice from Table 1 that the points associated with 0% correct differ across payoff matrices. Finally, we computed the deviation between the observed and optimal decision criterion standardized by category d’

\[ \text{decision criterion deviation} = k - k_o = \frac{\ln(\beta)}{d'} - \frac{\ln(\beta_o)}{d'} \]

Baseline Condition

The baseline condition was included to ensure that observers had accurate knowledge of the category distributions prior to any base-rate or cost-benefit manipulations. To determine whether this goal was achieved, we computed the accuracy, point, and decision criterion deviation measures along with 95% confidence intervals for the final block of trials separately for each level of category d’. For category d’ = 1.0, 2.2, and 3.2, respectively the accuracy deviations were \(-2.99 \pm 2.64, -3.19 \pm 1.80,\) and \(-3.59 \pm 2.03,\) the point deviations were \(-3.01 \pm 2.64, -3.07 \pm 1.80,\) and \(-3.24 \pm 2.04,\) and the decision criterion deviations were \(-0.08 \pm .14, .26 \pm .28,\) and \(.08 \pm .39.\) Notice that for the accuracy and point deviation measures the upper bound on the 95% confidence interval was approximately 1 unit below the optimal values. More importantly, for the decision criterion deviation measure the optimal decision criterion fell within the 95% confidence interval for all three category d’ levels.
For completeness, we also computed estimates of $d'$ and 95% confidence intervals for the final block of trials for each observer. For category $d' = 1.0, 2.2,$ and $3.2,$ respectively the estimated $d'$ values were $0.91 \pm 0.09, 2.08 \pm 0.13,$ and $3.31 \pm 0.70.$ In every case, the category $d'$ value $(1.0, 2.2,$ and $3.2)$ fell within a 95% confidence interval. Taken together, these analyses suggest that observers learned the category distributions, and the optimal decision criterion by the end of the baseline condition.

It is important to note that we are not arguing that the observer applied the optimal decision criterion perfectly on every trial. On the contrary, decision bound theory predicts that there will be some “noise” associated with the application of the decision criterion. Lee and Janke (1964; see also Kubovy & Healy, 1977; Kubovy, Rapoport, & Tversky, 1971) computed the number of static cutoff violations (SCV), as an estimate of “criterial noise”. As a link to this important literature we performed the same computations on our data. A detailed algorithm for computing the number of SCVs is outlined in Kubovy and Healy (1977). Briefly, one first determines the criterion or cutoff that best separates the “A” and “B” responses. This is the cutoff that contains the largest number of “A” responses on one side of the cutoff and “B” responses on the other side of the cutoff. The number of SCVs is the number of responses mis-predicted by this static cutoff—that is the number of “A” responses on the “B” side of the static cutoff plus the number of “B” responses on the “A” side of the cutoff. During the last block of trials, we observed 12.22%, 9.58%, and 2.78% SCVs for the category $d' = 1.0, 2.2,$ and $3.2$ conditions respectively. These results converge nicely with those from Lee and Janke (1964) who observed approximately 10% SCVs for $d = 1.5.$

### Experimental Conditions

The “flat-maxima” hypothesis predicts that performance should be closest to optimal in the category $d' = 2.2$ condition, farthest from optimal in the category $d' = 1.0$ condition, and intermediate in the category $d' = 3.2$ condition. As an initial test of this hypothesis we conducted an ANOVA on the accuracy, point, and decision criterion deviation measures outlined above. For all three measures the effect of category $d'$ was significant [accuracy deviation: $F(2, 10) = 27.32, p < .001$; point deviation: $F(2, 10) = 4.20, p < .05$; decision criterion deviation: $F(2, 10) = 11.25, p < .001$]. Bonferroni post hoc analyses indicated that performance differed significantly between the category $d' = 1.0$ and category $d' = 2.2$ and between category $d' = 1.0$ and category $d' = 3.2$ conditions ($p < .05$ for both the accuracy and decision criterion deviations, and $p < .10$ for the point deviation), but not between the category $d' = 2.2$ and category $d' = 3.2$ conditions ($p > .05$). The averages for the three measures are plotted in the top panel of Figure 5. Although the category $d' = 2.2$ and category $d' = 3.2$ conditions did not differ significantly, notice that the performance ordering predicted by the “flat-maxima” hypothesis is supported—that is, performance is closest to optimal for category $d' = 2.2,$ farthest from optimal for category $d' = 1.0,$ and intermediate for category $d' = 3.2.2.$ The accuracy, point, and decision criterion deviation measures by category $d'$ condition, base-rate/negative/non-negative cost condition, and block (averaged across observers) are presented in Table 3 – 5, respectively.

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Insert Table 3 – 5 and Figure 5 about here
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The COBRA hypothesis predicts that the decision criterion should be closer to optimal in the unequal base-rate condition than in the asymmetric payoff matrix conditions (see Figure 3). Maddox and Bohil (2000) found that the decision criterion was more sub-optimal when negative costs, as compared with non-negative costs, were included in the payoff matrix (and base-rates were equal). Taken together, COBRA and the Maddox and Bohil (2000) results predict that the decision criterion should be closest to optimal in the base-rate conditions, farthest from optimal in the asymmetric payoff matrix conditions with negative costs, and intermediate in the asymmetric payoff matrix conditions with non-negative costs. The same ordering is predicted for the point deviation measure since the optimal decision criterion maximizes expected reward. A different pattern of results is predicted for the accuracy deviation measure though. Because reward and accuracy maximization are incompatible when payoff matrices are asymmetric, the COBRA hypothesis predicts that the observer will be more accurate than the optimal classifier since the optimal classifier is willing to sacrifice accuracy in order to maximize reward. This effect should be magnified when the cost of an incorrect response is negative, as opposed to non-negative. Thus, we predict that the accuracy deviation measure should be positive (i.e., greater than optimal) for both asymmetric payoff conditions, but should be more positive in the negative cost conditions than in the non-negative cost conditions.

\(^2\) For completeness we also estimated $d'$ from signal detection theory and submitted these data to ANOVA. The only significant effect was the main effect of category discriminability. Bonferroni post hoc analyses suggested that the estimated $d'$ values were significantly different across all three pairs of category discriminabilities. These data are not discussed further.
Before testing these predictions we first compared performance between the two non-negative cost conditions (A and B) and between the two negative cost conditions (A and B). The performance differences were small and non-significant (p > .05), therefore these conditions were collapsed. It is worth mentioning at this point that the roughly equivalent performance for the negative cost(A) and negative cost(B) conditions provides some support for the assumption that the subjective utility functions is approximately linear. The optimal decision criterion is the same in the two negative cost conditions when the utility function is linear, but is different in the two negative cost conditions for a wide range of non-linear utility functions. If the utility function was highly non-linear then it is likely that performance would be very different in these two conditions, yet performance differences were not observed. Similarly, if the utility function was highly non-linear then it is likely that performance would be very different in the two non-negative cost conditions, yet no performance differences were observed. Although clearly not a definitive test of the linearity of the utility function, these results are suggestive.

An ANOVA was conducted on the three deviation measures to determine whether performance across the base-rate, non-negative cost, and negative cost conditions differed. There was a significant performance difference across conditions for the accuracy deviation measure \([F(2, 10) = 8.595, p < .01]\), but not for the point nor decision criterion deviation measures. The averages for the three measures are plotted in the middle panel of Figure 5. Notice that the accuracy deviation measure was positive for the two asymmetric payoff conditions, was larger for the negative cost condition than for the non-negative cost condition, and was negative for the base-rate condition, as predicted by the COBRA hypothesis and Maddox and Bohil (2000). Although non-significant, the decision criterion deviation measure yielded the predicted ordering with the most nearly optimal decision criteria resulting in the base-rate condition, and the least optimal resulting in the negative cost condition. The results for the point deviation measure were more equivocal, although again non-significant. Performance was closer to optimal for the non-negative than the negative cost conditions, as suggested by the Maddox and Bohil (2000) study, but was farthest from optimal in the base-rate condition, contrary to the predictions of COBRA. However, a closer examination of Table 4 suggests that the base-rate point deviation was largest only for category \(d' = 2.2\). For category \(d' = 1\) and category \(d' = 3.2\) the base-rate conditions yielded the smallest point deviations.

Finally, we examined the effects of learning on performance by conducting an ANOVA on the training/transfer block factor. The block effect was significant for the decision criterion deviation measure \([F(6, 30) = 7.33, p < .001]\), was marginally significant for the point deviation measure \([F(6, 30) = 2.054, p < .10]\) and was non-significant for the accuracy deviation measure (p > .05). Bonferroni post hoc analyses for the point and decision criterion deviation measures suggested that most of the learning occurred between the first and second blocks of trials. The block effects for the three measures are plotted in the bottom panel of Figure 5.

For completeness we also computed the percentage of SCVs. Interestingly, the percentage of SCVs decreased with category \(d'\) as observed for the baseline conditions although the difference were much smaller (8.61%, 8.22%, and 7.50% for category \(d' = 1.0, 2.2,\) and 3.2, respectively). There was also a monotonic decrease in the percentage of SCVs across negative cost (6.85%), non-negative cost (6.71%), and base-rate (5.37%) conditions. One reasonable, albeit post hoc, explanation for these results is that the prevalence of SCVs declines as optimal accuracy increases (i.e., with increasing category \(d'\)) and as the competition between reward and accuracy decreases (i.e., across cost-benefit/base-rate conditions).

Taken together these analyses provide strong initial support for the “flat-maxima” hypothesis, somewhat weaker support for the COBRA hypotheses, and for the effect of negative costs observed in Maddox and Bohil (2000). We turn now to the model-based analyses where we develop a series of decision bound models, each of which was applied simultaneously to the data from all experimental conditions separately by observer and block. A major goal of the model-based analyses is to test more fully the “flat-maxima” and COBRA hypothesis, and to develop and test a hybrid model that incorporates the assumptions of both hypotheses.

Simultaneous Model Fits of All Experimental Conditions

We begin this section with a brief overview of decision bound theory that provides the underlying framework for our modeling endeavor. The theory is described in detail in numerous articles (e.g., Ashby, 1992a; Ashby & Perrin, 1988; Ashby & Townsend, 1986; Maddox & Ashby, 1993). We then instantiate the “flat-maxima” and COBRA hypotheses within the framework of a decision bound model, and summarize the results of the model-based analyses.

Decision Bound Theory

Decision bound theory assumes that the observer attempts to respond optimally but is unable to because two sub-optimalties, perceptual noise and criterial noise, are inherent in all humans (and other organisms). Perceptual noise exists because there is trial-by-trial variability in the perceptual information associated with each stimulus. Criterial noise exists because there is trial-by-trial variability in the observer’s memory for the decision criterion. Because
perceptual and criterial noise exist, the human observer cannot attain the level of performance reached by the optimal classifier (i.e., cannot maximize long-run reward). Even so, decision bound theory assumes that the observer attempts to use the same strategy as the optimal classifier, but with less success due to the effects of perceptual and criterial noise. All of the models developed in this article acknowledge the existence of perceptual and criterial noise, and included parameters to account for their effects.

Besides perceptual and criterial noise, other sub-optimality might exist. For example, sub-optimality might exist in category distribution knowledge. All of the models tested in this article assumed that the observer had knowledge of the category structures. This was an important assumption because our interest was in studying observers’ decision criterion learning, and not potential sub-optimality in category distribution knowledge. To ensure that this was a reasonable assumption, the first session of the experiment was a baseline condition in which no base-rate or cost-benefit manipulation was present. As presented earlier, the observers’ decision criteria estimates were not significantly different from the optimal decision criterion, and their d’ estimates were not significantly different from the true values, suggesting that observers did have accurate knowledge of the category distributions. In addition, all observers completed a number of baseline trials at the beginning of each of the 15 experimental conditions to ensure accurate knowledge of the category structures, and to minimize any within-observer carry-over effects from one experimental condition to the next (see Methods).

Model Details

Each model was applied simultaneously to the data from all 15 experimental conditions separately by block and observer. During training, each block consisted of 60 experimental trials, and the observer was required to respond “A” or “B” for each stimulus. Since there were 15 conditions, each model was fit to a total of 1800 estimated response probabilities from each training block [60 trials x 2 response types (“A” or “B”) x 15 conditions]. During transfer a total of 120 trials were presented and so the model was fit to a total of 3600 estimated response probabilities. The model yielded predicted probabilities of responding “A” and “B”, but since P(“A”) = 1-P(“B”) there were 900 degrees of freedom in the training data, and 1800 degrees of freedom in the transfer data. Each model included three “noise” parameters whose values were estimated from the data, one for each of the three category d’ levels. The “noise” parameter represented the sum of perceptual and criterial noise (Ashby, 1992a; Maddox & Ashby, 1993). The model parameters were estimated using maximum likelihood procedures (Ashby, 1992b; Wickens, 1982).

The goal of the model-based analyses was to shed some light on the nature of the sub-optimality in decision criterion placement that result when base-rates are unequal and payoff matrices are asymmetric. In particular, we were interested in testing the validity of the “flat-maxima” and COBRA hypotheses. To facilitate the development of each model consider the following equation that determines the decision criterion used by the observer on i trials (k_i):

$$k_i = w_k_a + (1-w)k_r$$

(4)

where k_a is the decision criterion that maximizes expected accuracy (i.e., the equal likelihood decision criterion), k_r is the decision criterion used by the observer to maximize expected reward, and w is the importance (or weight) given to expected accuracy maximization. Recall that k_r = k_r when base-rates are unequal (and the payoff matrix is symmetric), but k_a when the payoff matrix is asymmetric (and base-rates are equal). We began by developing four models, each of which makes different assumptions about the k_a and w values. The nested structure of the models is presented in Figure 6. The number of free parameters (in addition to the three noise parameters described above) is presented in parentheses. The arrows point to the more general model. Models at the same level have the same number of free parameters.

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Insert Figure 6 about here

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The Optimal model instantiates neither the “flat-maxima”, nor the COBRA hypotheses. Specifically, it assumes that the decision criterion used by the observer to maximize expected reward is the optimal decision criterion (i.e., k_r = k_r), and that there is no competition between reward and accuracy maximization (i.e., w = 0). This model contains no additional free parameters beyond the three “noise” parameters.

The “Flat-Maxima” model instantiates the “flat-maxima” hypothesis, but not the COBRA hypotheses. Specifically, it assumes that the decision criterion used by the observer to maximize expected reward (k_r) is determined by the steep-ness of the ORF. A single steep-ness parameter is estimated from the data. This single steep-ness parameter determines three distinct decision criterion values for the three category d’ conditions (see Figure 2b). Because the decision criterion values are derived from the ORF, this model is constrained to predict that the decision criterion will be closest to optimal in the category d’ = 2.2 condition, farthest from optimal in the category d’ = 1.0 condition, and intermediate in the category d’ = 3.2 condition. In addition, this model assumes that there is no
competition between accuracy and reward maximization (i.e., \( w = 0 \)). Notice that this model contains one free parameter (in addition to the three “noise” parameters), and further that the Optimal model is a special case of the “Flat-Maxima” model in which the steepness value is equal to zero (i.e., \( k_a = k_o \)).

The COBRA model instantiates the COBRA hypothesis but not the “flat-maxima” hypothesis. Specifically, it assumes that the decision criterion used by the observer to maximize expected reward is the optimal decision criterion (i.e., \( k_a = k_o \)), but allows for a competition between reward and accuracy maximization by estimating the Equation 4 \( w \) parameter from the data. Notice that this model contains one free parameter (in addition to the three “noise” parameters), and that the Optimal model is a special case of the COBRA model in which \( w = 0 \).

The Hybrid\((w)\) model instantiates both the “flat-maxima”, and the COBRA hypotheses. Specifically, it assumes that \( k_a \) is determined by the steepness of the ORF, and that there is a competition between accuracy and reward maximization. Notice that this model contains two free parameters (in addition to the three “noise” parameters), and also includes the latter three models as special cases.

A fifth, and final, model was also developed. The Hybrid\((w_-; w+)\) model was developed to test the hypothesis proposed by Maddox and Bohil (2000) that the weight placed on accuracy maximization was greater in negative cost, as compared with non-negative cost conditions. In this model two \( w \) parameters were estimated. The \( w_- \) parameter was applied to the data from the negative cost conditions, and the \( w_+ \) parameter was applied to the data from the non-negative cost conditions. This model includes all other models as special cases.

Because the model parameters were estimated using maximum likelihood procedures and because the models either have the same number of parameters, or are nested, we were able to use Likelihood Ratio (\( G^2 \)) tests to identify the model that provided the most parsimonious account of the data (Ashby, 1992b; Wickens, 1982). The most parsimonious model (assuming an alpha level of .05) for each observer by block is presented in Table 6. Several comments are in order. First, notice that the Hybrid\((w_-; w+)\) model provided the most parsimonious account of the data for at least 5 of the 7 blocks of trials for Observers 1, 3, 5, and 6. This result provides support for the hypothesis that the steepness of the ORF, and a competition between reward and accuracy are important components of decision criterion placement for these observers. In addition, it suggests that the presence of negative vs. non-negative costs affect the importance placed on accuracy maximization. Second, the Hybrid\((w)\) model provided the most parsimonious account of the data for 6 of the 7 blocks of trials for Observer 2. This result again suggests that the steepness of the ORF, and a competition between reward and accuracy are important components of decision criterion placement, but for this observer allowing separate weights for negative and non-negative costs did not provide a significant improvement in fit. Finally, the performance of Observer 4 is quite interesting. For this observer the Hybrid\((w_-; w+)\) model dominated during the early blocks of trials, but during that latter blocks the COBRA model dominated. One interpretation of this result is that the observer’s expected reward decision criterion, \( k_a \), was sub-optimal during the early blocks of trials, and that the observer weighted accuracy maximization more heavily when negative costs were present. However, as the observer gained experience with the task (i.e., during the latter blocks), the expected reward decision criterion approached the optimal value (i.e., \( k_a = k_o \)), and the competition was between this optimal expected reward decision criterion, and the equal likelihood criterion.

Because the models were applied separately to each block of trials, we can examine the parameter values to determine how they changed as the observers’ gained experience with the task. Although the most parsimonious model differed across observers and blocks of trials, it seems reasonable to examine the parameters of the Hybrid\((w_-; w+)\) model since it instantiates all of the relevant hypotheses (“flat-maxima”, COBRA, and the effects of negative/non-negative costs) and provided the most parsimonious account of the data more than half of the time (26 of 42 cases: 6 observers x 7 blocks).

Figure 7a plots the steepness value (averaged across observers) from the Hybrid\((w_-; w+)\) model for each block of trials. Figure 7b plots the accuracy weights (averaged across observers) for the negative (\( w_- \)) and non-negative (\( w_+ \)) cost conditions from the same model for each block of trials. A one-way ANOVA was conducted on the steepness values revealing a significant effect of block \( [F(6,30)= 6.841, p < .001] \). The steepness values decreased rapidly toward the optimal steepness of zero across the first three blocks, and stabilized throughout the remaining blocks of trials. This pattern also held at the level of the individual observer. The steepness value was smaller in block 2 than block 1, and smaller in block 3 than block 2 for 5 and 6 of the observers, respectively.

The \( w_+ \) values changed little across blocks remaining stable at a value around .30. A one-way ANOVA revealed that the \( w_+ \) values did not differ significantly across blocks (\( p > .05 \)). This pattern was mirrored in the
individual observer data where the w+ values were smaller during blocks 6 and 7 than block 1 for only half of the observers. The w- values were large during the first block of trials (around .60) then gradually declined toward the w+

values. A one-way ANOVA revealed a marginally significant block effect \[F(6, 30) = 2.28, p = .06\]. Because the w- values were highly variable during the early blocks, we decided to compare the w- values during early learning (i.e., collapsed across blocks 1 – 3) with those during late learning (i.e., collapsed across blocks 4 – 7). The effect was again marginally significant \[F(1, 5) = 4.575, p = .09\], suggesting that the weight placed on accuracy maximization declined with learning when the cost of an incorrect response was negative. This pattern was also observed in the individual observer data where the w- values were smaller during blocks 6 and 7 than during block 1 for 5 of the 6 observers.

In summary, these analyses suggest that the effects of category d’, base-rate/cost-benefits, and the presence of negative vs. non-negative costs on observer’s decision criterion learning was best captured by a hybrid model that incorporates both the “flat-maxima” and COBRA hypotheses. The model parameters indicated that the observer’s expected reward decision criterion, \(k_o\), shifted quickly toward the optimal criterion, \(k_o\), during the first 200 trials (3 blocks), and then changed more gradually with extended training. The model parameters also indicated that there was little change over blocks in the weight assigned to accuracy maximization when the cost of an incorrect response was non-negative, but that there was a decline in the weight assigned to accuracy maximization when the cost of an incorrect response was negative. Finally, by the end of training, the weight assigned to accuracy maximization was still substantial (around .30 for w+ and w-), but differed little across negative and non-negative cost conditions.

General Discussion

This article reports the results of an experiment that examined the effects of category discriminability, base-rate ratio, and the presence of negative vs. non-negative costs in asymmetric payoff matrices on decision criterion learning in a simulated medical diagnosis task. Three levels of category discriminability (category \(d’ = 1, 2.2, \) and 3.2) were combined factorially with five base-rate/cost-benefit conditions for a total of 15 experimental conditions. These included a 3:1 base-rate condition, two negative cost conditions, and 2 non-negative cost conditions (see Table 2). Each observer completed several blocks of trials in each of the 15 experimental conditions allowing us to capture the time course of decision criterion learning across conditions, and providing a rich database for current and future model testing.

Analyses of points, accuracy, and decision criterion estimates derived from signal detection theory indicated that performance was closest to that predicted by the optimal classifier for category \(d’ = 2.2\), was intermediate for category \(d’ = 3.2\), and was farthest from optimal for category \(d’ = 1.0\). In addition, these same measures indicated that performance was closest to optimal when base-rates were unequal, was intermediate when cost-benefits were manipulated and no point loss obtained for an incorrect response (non-negative cost), and was farthest from optimal when cost-benefits were manipulated and a point loss was obtained for an incorrect response (negative cost).

The theoretical analyses focused on a series of decision bound models, each of which was applied simultaneously to the data from all experimental conditions separately by observer and block. A major goal of the model-based analyses was to provide a strong test of the “flat-maxima” and COBRA hypotheses, and to develop and test a hybrid model that incorporated the assumptions of both hypotheses. The “flat-maxima” hypothesis suggests that the decision criterion used by the observer to maximize expected reward is determined by the steep-ness of the ORF (see Figure 2). The COBRA hypothesis suggests that observers place some importance (or weight) on reward and accuracy maximization. When both goals can be achieved simultaneously, as in unequal base-rate conditions (see Figure 3a), there is effectively no competition, and the decision criterion approaches that used by the optimal classifier. When both goals cannot be achieved simultaneously, as in asymmetric payoff (unequal cost-benefit) conditions, the competition leads to the use of a decision criterion intermediate between the optimal decision criterion, and the decision criterion that maximizes expected accuracy.

As a test of the “flat-maxima” and COBRA hypotheses, five models were fit to the data. One model assumed that both the “flat-maxima” and COBRA hypothesis were unsupported. The second model instantiated the “flat-maxima” hypothesis but not the COBRA hypothesis. The third model instantiated the COBRA hypothesis but not the “flat-maxima” hypothesis. The fourth model was a hybrid model that incorporated both the “flat-maxima” and COBRA hypotheses. The fifth model generalized the former hybrid model by allowing the importance (or weight) placed on accuracy maximization to differ across asymmetric payoff matrices that included either negative or non-negative costs.

Some individual differences were observed, but, in general, the hybrid model that allowed for different accuracy
weights across negative and non-negative cost conditions provided the best account of the data indicating that model-based instantiations of both the “flat-maxima” and COBRA hypotheses were required to account for decision criterion learning. The model parameters suggested the following. First, learning of the reward-maximization decision criterion was rapid during the first 200 trials or so, then slowed considerably during the remaining trials, gradually approaching the optimal decision criterion. Second, the weight placed on accuracy maximization in the negative cost conditions was large early in learning but decreased consistently across blocks of trials. Third, the weight placed on accuracy maximization in the non-negative cost conditions was much smaller early in learning, relative to that for the negative cost conditions, and remained relatively stable across learning. Fourth, by the final block of trials, the weight placed on accuracy maximization in the negative and non-negative cost conditions was approximately equal. Thus, as predicted by the “flat-maxima” hypothesis, it appears that the primary effect of category discriminability is on the observer’s placement of the reward-maximizing decision criterion, and the placement of the reward-maximizing decision criterion is unaffected by base-rate/cost-benefit condition. Cost-benefit manipulations affect the emphasis placed on accuracy maximization, but the effect is quite different for cases in which the cost of an incorrect response is negative, as compared with cases in which the cost of an incorrect response is non-negative. There appears to be a strong emphasis on accuracy maximization early in learning when the cost of an incorrect response is negative. This “bias” toward accuracy maximization decreases with experience, but only partially. When the cost of an incorrect response is non-negative, on the other hand, the observer shows a smaller “bias” toward accuracy maximization that remains fairly constant over blocks. Interestingly, by the final block of trials the weight placed on accuracy maximization is approximately equal for negative and non-negative cost conditions.

**Category Discriminability and the “Flat-Maxima” Hypothesis**

The “flat-maxima” hypothesis predicts a nonlinear relationship between performance and category \( d' \) such that performance should be best for category \( d' = 2.2 \), worst for category \( d' = 1.0 \), and intermediate for category \( d' = 3.2 \). We found support for this ordering in three performance measures (accuracy, point, and decision criterion), and in a rigorous model-based instantiation of the “flat-maxima” hypothesis. Lee and Zentall (1966) found a monotonic relationship between category \( d' \) (category \( d' \) values of 1.25, 2.25, and 3.25) and several measures of performance. Thus, Lee and Zentall’s (1966) findings converge with the predictions of the “flat-maxima” hypothesis and the results of the current study for a comparison of category \( d' = 1.0 \) with category \( d' = 2.2 \), but diverge for a comparison of category \( d' = 2.2 \) with category \( d' = 3.2 \). Lee and Zentall (1966) found a performance increase from category \( d' = 2.2 \) to category \( d' = 3.2 \), whereas the “flat-maxima” hypothesis predicts a decrease, as observed in the current study. There are several possible explanations for these inconsistent results. First, it is possible that the performance differences between category \( d' = 1.0 \) and category \( d' = 2.2 \) are robust, whereas the differences between category \( d' = 2.2 \) and 3.2 may be less reliable. In support of this hypothesis, the observed performance differences between category \( d' = 1.0 \) and category \( d' = 2.2 \) were larger than the performance differences observed between category \( d' = 2.2 \) and category \( d' = 3.2 \) in both Lee and Zentall and the current study. In fact, the performance differences between category \( d' = 2.2 \) and 3.2 were non-significant in the current study (no significance tests were offered in Lee and Zentall). A second possibility is that the nature of the category \( d' \) manipulation might affect the performance difference observed between category \( d' = 2.2 \) and category \( d' = 3.2 \). In the current study, the category variances were held fixed and the mean separation was increased to increase category \( d' \). In the Lee and Zentall (1966) study on the other hand, category \( d' \) was increased by simultaneously increasing mean separation and decreasing category variance. This latter approach was taken because the range of possible stimulus values was limited. Although either of these (or other) possibilities is reasonable, our feeling is that the stronger evidence favors the “flat-maxima” hypothesis over the monotonic relation observed by Lee and Zentall (1966). Lee and Zentall’s (1966) results are based on aggregate data. Aggregate data are susceptible to a number of potential problems not the least of which are the presence of outliers. The model-based analysis of single observer performance in the current study alleviates this problem, and provides strong support for the predictions of the “flat-maxima” hypothesis. Even so, more work is needed to determine the true performance relation between category \( d' = 2.2 \) and category \( d' = 3.2 \) conditions.

**COBRA and the Effects of Negative vs. Non-negative Costs**

Unequal base-rates led each observer to use a decision criterion that was closer to the optimal decision criterion than when payoffs were asymmetric. This replicates the results of a number of studies and extends the finding to a wide range of category \( d' \) values (e.g., Bohil & Maddox, in press; Healy & Kubovy, 1981; Maddox & Bohil, 1998a). Similarly, non-negative costs led the observer to use a decision criterion that was closer to the optimal decision criterion than when costs were negative. This latter result is inconsistent with the probability learning literature in which the presence of negative costs generally leads to faster learning (e.g., Siegel & Goldstein, 1959; Bereby-Meyer & Erev, 1998). However, there are several procedural differences between categorization and probability learning tasks that
might account for this performance difference. In both tasks, the observer must choose between one of two possible outcomes. In categorization, the observer uses stimulus information to help choose a response. The optimal strategy is to compute the category likelihood ratio for the presented stimulus and compare that to the optimal decision criterion determined from the base-rate and payoff matrix entries. In probability learning, on the other hand, no stimulus is presented. In probability learning the base-rates of the two outcomes are unequal, and the optimal strategy is to choose the most likely outcome on every trial. This is effectively a categorization problem with no stimulus information, and in which the optimal decision criterion is infinite. The lack of a stimulus for encoding and processing is likely a critical difference between probability learning and categorization that results in very different cognitive processing demands. For example, recent neuropsychological models of category learning (Ashby, Alfonso-Reese, Turken, & Waldron, 1998) postulate that sub-cortical structures in the basal ganglia are involved in stimulus representation, and provide input to response selection modules in frontal cortex. In a probability learning task there is no stimulus and thus much of the early processing that is critical to category learning is absent.

Another possibility is that the effect of negative/non-negative costs observed in the current study is not due to the negative/non-negative distinction but rather to some other difference between the relevant payoff matrices. Erev (1998) offers one possibility. Erev (1998; see also Bereby-Meyer & Erev, 1998) suggests that learning slows as payoff variance increases. In the current study the non-negative cost payoff matrices are characterized by smaller payoff variance than the negative cost payoff matrices, thus confounding the negative/non-negative cost distinction with the payoff variance distinction. Future work is needed to answer this by including conditions in which payoff variance is held constant across negative and non-negative cost conditions.

Model-Based COBRA Implementation

In this article we implemented the COBRA hypothesis by assuming that the observer differentially weighted the expected accuracy and reward-maximizing decision criteria and applied an intermediate decision criterion on every trial. Clearly other instantiations are possible. In this section we entertain an alternative hypotheses that postulates a single decision criterion (perhaps driven by the steepness of the ORF) but a nonlinear relation between subjective utility and the objective values, u(V_{ij}). Suppose that there is a covert penalty associated with errors such that u_{ab} = V_{ab} - \varepsilon and u_{A} = V_{BA} - \lambda. One possibility is that \varepsilon = \lambda, but that these values differ across negative and non-negative cost conditions. This can account qualitatively for the sub-optimal decision criterion learning in asymmetric payoff conditions, and the negative/non-negative cost effect if the \varepsilon and \lambda values are positive, and the \varepsilon and \lambda values for the negative cost condition are at least twice as large as those for the non-negative cost condition. This is clearly a reasonable alternative to the formulation tested in the model-based analysis section. One weakness of this approach is that in many cases \varepsilon \neq \lambda. For example, the covert penalty of misdiagnosing a non-malignant tumor as cancerous may be less than the covert penalty of misdiagnosing a malignant tumor as non-cancerous. It would be straightforward to relax this assumption, but this would require two additional free parameters, whereas the formulation offered in the current study requires only two free parameters. Even so, a comparison of these alternative formulations provides a fertile ground for future research.

Relations to Other Models

It is important to acknowledge that the models proposed in this article are not “learning” models in the typical sense. The current models assume that the observer uses a static decision criterion in the presence of criterial noise. The models provide information about “average” performance within a block of trials, and by applying the models separately to each block of trials they provide information about the effect of learning. This is a useful approach because changes in parameter values across blocks of trials provide useful information regarding the “flat-maxima” and COBRA hypotheses. Other models have been proposed in the literature that model trial-by-trial changes in the decision criterion. These include, among others, Busemeyer and Myung’s (1992) Hill-Climbing model, Erev’s (1998) Criterion Reinforcement Learning (CRL) model (see also Wallsten & Gonzalez-Vallejo’s, 1994 Stochastic Judgment Model). These models have important similarities and differences that are beyond the scope of this article. For illustrative purposes we focus the current discussion on Erev’s (1998) CRL model. In short, the CRL model assumes that the observer’s decision criterion can change from trial to trial and is affected by the costs and benefits, base-rates, etc. Erev (1998) showed that the CRL model predicts ex ante many of the performance regularities that have been observed in binary probabilistic classification studies, and that these predictions are robust across a wide range of parameter values. Erev (personal communication, November, 2000) applied the CRL model to the task presented in this article and captured the main findings: in particular, the non-linear effect of category d’ and the more sub-optimal performance for negative cost conditions.

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3 We are indebted to an anonymous reviewer of an earlier version of this article for suggesting this interesting alternative.
Our feeling is that both approaches have merit. The approach taken in this article focuses on a rigorous description of the empirical effects, and instantiates simple hypotheses to account for these at the block-by-block level. The CRL and other learning models make trial-by-trial predictions, and at least with respect to the CRL model, can account for many empirical results ex ante.

Summary

In conclusion, the present study extends our understanding of decision criterion learning when base-rates, cost-benefits, and category discriminability are manipulated. Strong support for the “flat-maxima” and COBRA hypotheses were obtained in “aggregate” performance as well as in model-based analyses of individual observer performance. A new model of base-rate and cost-benefit learning is proposed and tested that incorporates simultaneously the “flat-maxima” and COBRA hypotheses. The model provided a good account of the data, and yielded parameter values that were readily interpretable, and offered important insights into the psychological processes involved in decision criterion learning.

References


Table 1. Category Base-rates and Costs-Benefits for Each of the Five Experimental Conditions

<table>
<thead>
<tr>
<th>Category</th>
<th>Base-rate</th>
<th>Cost-Benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P(A)</td>
<td>P(B)</td>
</tr>
<tr>
<td>3:1 Base-rate</td>
<td>.75</td>
<td>.25</td>
</tr>
<tr>
<td>Non-negative Cost(A)</td>
<td>.50</td>
<td>.50</td>
</tr>
<tr>
<td>Non-negative Cost(B)</td>
<td>.50</td>
<td>.50</td>
</tr>
<tr>
<td>Negative Cost(A)</td>
<td>.50</td>
<td>.50</td>
</tr>
<tr>
<td>Negative Cost(B)</td>
<td>.50</td>
<td>.50</td>
</tr>
</tbody>
</table>

Table 2. Optimal Points (per 60 trial block), Accuracy and β values

<table>
<thead>
<tr>
<th>Category d' = 1</th>
<th>Category d' = 2.2</th>
<th>Category d' = 3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points</td>
<td>Accuracy</td>
<td>Points</td>
</tr>
<tr>
<td>-----------------</td>
<td>-------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>Baseline</td>
<td>83  69.2</td>
<td>103  85.9</td>
</tr>
<tr>
<td>3:1 Base-rate</td>
<td>93  77.8</td>
<td>106  88.7</td>
</tr>
<tr>
<td>Non-negative Cost(A)</td>
<td>93  61.0</td>
<td>106  82.9</td>
</tr>
<tr>
<td>Non-negative Cost(B)</td>
<td>153  61.0</td>
<td>166  82.9</td>
</tr>
<tr>
<td>Negative Cost(A)</td>
<td>67  61.0</td>
<td>93  82.9</td>
</tr>
<tr>
<td>Negative Cost(B)</td>
<td>67  61.0</td>
<td>93  82.9</td>
</tr>
</tbody>
</table>
Table 3: Standardized Deviation from Optimal Accuracy (Averaged Across Observers)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Block</th>
<th>Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Non-negative</td>
<td>8.40</td>
<td>6.35</td>
</tr>
<tr>
<td>Negative</td>
<td>7.03</td>
<td>5.44</td>
</tr>
<tr>
<td>Base-rate</td>
<td>-9.61</td>
<td>-4.25</td>
</tr>
<tr>
<td>Weighted Ave.</td>
<td>4.25</td>
<td>3.87</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Condition</th>
<th>d'=2.2</th>
<th>Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-negative</td>
<td>0.99</td>
<td>-2.03</td>
</tr>
<tr>
<td>Negative</td>
<td>1.99</td>
<td>0.65</td>
</tr>
<tr>
<td>Base-rate</td>
<td>-5.38</td>
<td>-6.01</td>
</tr>
<tr>
<td>Weighted Ave.</td>
<td>0.11</td>
<td>-1.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Condition</th>
<th>d'=3.2</th>
<th>Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-negative</td>
<td>-3.87</td>
<td>-2.53</td>
</tr>
<tr>
<td>Negative</td>
<td>-3.13</td>
<td>-2.68</td>
</tr>
<tr>
<td>Base-rate</td>
<td>-3.37</td>
<td>-2.50</td>
</tr>
<tr>
<td>Weighted Ave.</td>
<td>-3.47</td>
<td>-2.59</td>
</tr>
</tbody>
</table>

Note: Standardized Deviation from Optimal Accuracy = (Observed Accuracy - Optimal Accuracy)/(Optimal Accuracy – 0). The weighted average is used because two payoff matrices are associated with the Non-negative condition, two payoff matrices are associated with the Negative condition, but only one Base-rate condition is used.
<table>
<thead>
<tr>
<th>Condition</th>
<th>Block</th>
<th>Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Non-negative</td>
<td>-6.03</td>
<td>-7.02</td>
</tr>
<tr>
<td>Negative</td>
<td>-11.75</td>
<td>-7.02</td>
</tr>
<tr>
<td>Base-rate</td>
<td>-9.61</td>
<td>-4.25</td>
</tr>
<tr>
<td>Weighted Ave.</td>
<td>-9.03</td>
<td>-6.46</td>
</tr>
<tr>
<td>d'=1</td>
<td>Non-negative</td>
<td>-0.91</td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td>-1.69</td>
</tr>
<tr>
<td></td>
<td>Base-rate</td>
<td>-5.38</td>
</tr>
<tr>
<td></td>
<td>Weighted Ave.</td>
<td>-2.12</td>
</tr>
<tr>
<td>d'=2.2</td>
<td>Non-negative</td>
<td>-4.46</td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td>-4.10</td>
</tr>
<tr>
<td></td>
<td>Base-rate</td>
<td>-3.37</td>
</tr>
<tr>
<td></td>
<td>Weighted Ave.</td>
<td>-4.10</td>
</tr>
<tr>
<td>d'=3.2</td>
<td>Non-negative</td>
<td>-3.80</td>
</tr>
</tbody>
</table>

Note: Standardized Deviation from Optimal Points = (Observed Points - Optimal Points)/(Optimal Points – Predicted Points for 0% Accuracy). The weighted average is used because two payoff matrices are associated with the Non-negative condition, two payoff matrices are associated with the Negative condition, but only one Base-rate condition is used.
### Table 5. Standardized Deviation from Optimal Decision Criterion (Averaged Across Observers)

<table>
<thead>
<tr>
<th>Condition</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>d’=1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-negative</td>
<td>-0.71</td>
<td>-0.68</td>
<td>-0.43</td>
<td>-0.57</td>
<td>-0.60</td>
<td>-0.55</td>
<td>-0.45</td>
<td>-0.57</td>
</tr>
<tr>
<td>Negative</td>
<td>-0.96</td>
<td>-0.71</td>
<td>-0.73</td>
<td>-0.51</td>
<td>-0.63</td>
<td>-0.41</td>
<td>-0.61</td>
<td>-0.65</td>
</tr>
<tr>
<td>Base-rate</td>
<td>-0.87</td>
<td>-0.34</td>
<td>0.04</td>
<td>0.04</td>
<td>-0.08</td>
<td>0.45</td>
<td>-0.07</td>
<td>-0.12</td>
</tr>
<tr>
<td>Weighted Ave.</td>
<td>-0.84</td>
<td>-0.62</td>
<td>-0.45</td>
<td>-0.42</td>
<td>-0.51</td>
<td>-0.29</td>
<td>-0.44</td>
<td>-0.51</td>
</tr>
<tr>
<td><strong>d’=2.2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-negative</td>
<td>-0.08</td>
<td>0.22</td>
<td>-0.11</td>
<td>-0.04</td>
<td>0.05</td>
<td>0.14</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>Negative</td>
<td>-0.29</td>
<td>-0.11</td>
<td>-0.04</td>
<td>-0.12</td>
<td>0.11</td>
<td>0.06</td>
<td>-0.01</td>
<td>-0.06</td>
</tr>
<tr>
<td>Base-rate</td>
<td>-0.73</td>
<td>-0.33</td>
<td>-0.16</td>
<td>-0.10</td>
<td>0.24</td>
<td>-0.36</td>
<td>-0.05</td>
<td>-0.21</td>
</tr>
<tr>
<td>Weighted Ave.</td>
<td>-0.29</td>
<td>-0.02</td>
<td>-0.09</td>
<td>-0.08</td>
<td>0.11</td>
<td>0.01</td>
<td>0.01</td>
<td>-0.05</td>
</tr>
<tr>
<td><strong>d’=3.2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-negative</td>
<td>-0.21</td>
<td>-0.27</td>
<td>-0.37</td>
<td>-0.23</td>
<td>-0.16</td>
<td>-0.26</td>
<td>-0.18</td>
<td>-0.24</td>
</tr>
<tr>
<td>Negative</td>
<td>-0.25</td>
<td>-0.12</td>
<td>-0.20</td>
<td>-0.13</td>
<td>-0.10</td>
<td>-0.27</td>
<td>-0.10</td>
<td>-0.17</td>
</tr>
<tr>
<td>Base-rate</td>
<td>-0.39</td>
<td>-0.30</td>
<td>0.23</td>
<td>-0.11</td>
<td>-0.10</td>
<td>-0.10</td>
<td>-0.34</td>
<td>-0.16</td>
</tr>
<tr>
<td>Weighted Ave.</td>
<td>-0.26</td>
<td>-0.22</td>
<td>-0.18</td>
<td>-0.17</td>
<td>-0.12</td>
<td>-0.23</td>
<td>-0.18</td>
<td>-0.19</td>
</tr>
<tr>
<td><strong>Ave.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-negative</td>
<td>-0.33</td>
<td>-0.24</td>
<td>-0.30</td>
<td>-0.28</td>
<td>-0.24</td>
<td>-0.22</td>
<td>-0.19</td>
<td>-0.26</td>
</tr>
<tr>
<td>Negative</td>
<td>-0.50</td>
<td>-0.31</td>
<td>-0.32</td>
<td>-0.25</td>
<td>-0.21</td>
<td>-0.21</td>
<td>-0.24</td>
<td>-0.29</td>
</tr>
<tr>
<td>Base-rate</td>
<td>-0.67</td>
<td>-0.32</td>
<td>0.04</td>
<td>-0.05</td>
<td>0.02</td>
<td>0.00</td>
<td>-0.15</td>
<td>-0.16</td>
</tr>
<tr>
<td>Weighted Ave.</td>
<td>-0.47</td>
<td>-0.29</td>
<td>-0.24</td>
<td>-0.22</td>
<td>-0.17</td>
<td>-0.17</td>
<td>-0.20</td>
<td>-0.25</td>
</tr>
</tbody>
</table>

**Note:** Standardized Deviation from Optimal Decision Criterion = \( k - k_o = \ln(\beta)/d' - \ln(\beta_o)/d' = \ln(\beta/\beta_o)/d' \). The weighted average is used because two payoff matrices are associated with the Non-negative condition, two payoff matrices are associated with the Negative condition, but only one Base-rate condition is used.

### Table 6. Most Parsimonious Model by Observer and Block.

<table>
<thead>
<tr>
<th>Obs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w-; w+)</td>
<td>Optimal</td>
<td>Hybrid (w-; w+)</td>
<td>COBRA</td>
<td>Hybrid (w-; w+)</td>
</tr>
<tr>
<td>2</td>
<td>Hybrid (w)</td>
<td>Hybrid (w)</td>
<td>Hybrid (w)</td>
<td>&quot;Flat-Maxima&quot;</td>
<td>Hybrid (w)</td>
<td>Hybrid (w)</td>
<td>Hybrid (w)</td>
</tr>
<tr>
<td>3</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w)</td>
<td>COBRA</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w-; w+)</td>
</tr>
<tr>
<td>4</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w-; w+)</td>
<td>COBRA</td>
<td>COBRA</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w-; w+)</td>
<td>COBRA</td>
</tr>
<tr>
<td>5</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w)</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w)</td>
<td>Hybrid (w-; w+)</td>
</tr>
<tr>
<td>6</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w-; w+)</td>
<td>COBRA</td>
<td>Hybrid (w-; w+)</td>
<td>Hybrid (w)</td>
<td>Hybrid (w)</td>
<td>Hybrid (w-; w+)</td>
</tr>
</tbody>
</table>
Figure 1. Hypothetical distributions for categories A and B when category discriminability (d’) is equal to (a) 1.0, (b) 2.2, and (c) 3.2. The $\beta_0 = 1$ decision criterion denotes the criterion that is optimal when the base-rates are equal and the payoff matrix is symmetric. This is also referred to as the equal likelihood decision criterion. The $\beta_0 = 3$ decision criterion denotes the criterion that is optimal when there is a 3:1 base-rate ratio, or when the payoff matrix is asymmetric with a 3:1 cost-benefit ratio.
Figure 2. (a) Expected reward as a function of the decision criterion (relative to the optimal decision criterion; i.e., $k - k_o$), called the Objective Reward Function (ORF) for category discriminability, $d' = 1.0$, 2.2, and 3.2. (b) Steep-ness of the ORF (from panel a) for category discriminability, $d' = 1.0$, 2.2, and 3.2. The three vertical lines denote the $k - k_o$ values for a fixed value of steep-ness. Notice that the deviation from the optimal decision criterion is smallest for category $d' = 2.2$, is farthest from optimal for category $d' = 1.0$, and is intermediate for category $d' = 3.2$. 
a) \[ P(A) = 3 P(B) \]
\[(V_{aA} - V_{bA}) = (V_{bB} - V_{aB})\]

Figure 3. Schematic illustration the COmpetition Between Reward and Accuracy (COBRA) hypothesis. The \( k_r \) decision criterion denotes the decision criterion that is being used by the observer in an attempt to maximize expected reward. The \( k_a \) decision criterion denotes the decision criterion that maximizes expected accuracy. The \( k_1 \) decision criterion denotes the decision criterion resulting from the COBRA hypothesis with the assumption that less importance (or weight; \( w < .5 \)) is being placed on accuracy maximization. The \( k_2 \) decision criterion denotes the decision criterion resulting from the COBRA hypothesis with the assumption that more importance (or weight; \( w > .5 \)) is being placed on reward maximization.

b) \[ P(A) = P(B) \]
\[(V_{aA} - V_{bA}) = 3 (V_{bB} - V_{aB})\]

\[ k = wk_a + (1-w) k_r \]
Figure 4. Schematic illustration of a typical stimulus.
Figure 5. Accuracy, point, and decision criterion deviation values (averaged across observers) for the category d’ conditions (top panel), cost-benefit/base-rate conditions (middle panel), and block (bottom panel).
Note: All models listed below assume three noise parameters.

Figure 6. Nested relationship among the decision criterion models applied simultaneously to the data from all experimental conditions. The arrows point to a more general model. (see text for details).
Figure 7. (a) Steepness value (averaged across observers) from the Hybrid($w-$; $w+$) model for each block of trials. (b) Accuracy weights (averaged across observers) for the negative ($w-$; broken line) and non-negative ($w+$; solid line) cost conditions from the Hybrid($w-$; $w+$) model for each block of trials.