We describe three human spatial navigation experiments that investigate the role of perception, memory, uncertainty and decision strategy on human spatial navigation performance. These studies were conducted in virtual reality indoor environments that were visually impoverished. To better understand the effect of these variables on human navigation performance, we developed an ideal navigator model for indoor navigation based on Partially Observable Markov Decision Process (Cassandra, Kaelbling, & Littman, 1994; Chung, 1960; Kaelbling, Littman, & Cassandra, 1998). The model is designed to navigate through visually sparse indoor environments where multiple states (positions and orientations) in the environment can produce the same visual percept. The model is designed to minimize the number of actions (translations and rotations) required to move from an unknown starting state to a specific goal state within the environment. We used the model to compute human navigation efficiency by computing the ratio of the number of actions required by the ideal navigator relative to the number of actions taken by the human subjects. Experiment 1 investigated the effect of increasing the layout size on spatial way-finding efficiency and found that subjects’ efficiencies decreased as layout size increased. We investigated whether this reduction in navigation efficiency was due to visual perception (Experiment 2), memory, uncertainty or decision strategy (Experiment 3). The results revealed a significant reduction in efficiency as the layout size increased. By contrast, reducing visual information did not have a consistent effect on navigation efficiency. Results from Experiment 3 suggest that the inefficiencies in Experiment 1 are due to inefficiencies in the subject’s spatial updating strategy.
Introduction

After starting a new job in a new building, you need to develop an understanding of frequently visited locations within your new environment. Locations like the closest restroom to your office, the stairwell, the mailroom, the photocopier room and other locations all may serve as important places in your everyday activities. Rather than studying a map, someone may show you where some of these locations are, or you may simply find them through your own exploration. After a period of time (perhaps a few weeks) you typically have woven these experiences into an internal representation of the space that allows you to move effectively from one location to another. This internal representation is usually referred to as a cognitive map (Hirtle & Heidorn, 1993; Kuipers, 2001; O'Keefe & Nadel, 1978; Tolman, 1948).

The concept of a cognitive map has influenced how researchers describe the ultimate representation that we obtain after extensive exploration of a large-scale space. However, Tolman’s (1948) introduction of the concept did not provide an explicit description of how we generate this cognitive map is generated from individual experiences or what information is made explicit in the map. Furthermore, it is not clear how we access this information while navigating through a familiar environment.

Later research by Siegel and White (1975) provided a description of the metamorphosis of a cognitive map when a person becomes familiar with a specific environment. According to Siegel and White, a cognitive map begins as a set of landmarks. This is followed by a representation that makes explicit a sequence of actions needed to get from one location to another called routes. Finally, these sequences of actions (or routes) are united into an internal representation of the environment that is more map-like, called a survey representation. This survey representation is useful because it can be used to generate novel routes between two locations within the environment.

Although Siegel and White suggest that there are a sequence of stages in the development of a cognitive map a clear understanding of the specific processes underlying the acquisition and access of that map remains uncertain. It should be noted that there has been a great deal of research into what information is stored in the cognitive map (Cohen & Schuepfer, 1980; Franz, Schölkopf, Mallot, & Bülthoff, 1998; Gillner & Mallot, 1998; Hirtle & Hudson, 1991; Hirtle & Jonides, 1985; Hirtle & Kallman, 1988; Hirtle & Mascolo, 1986; Jacobs, Thomas, Laurance, & Nadel, 1998; Klitzky, Loomis, Golledge, Cicinelli, & et al., 1990; Kuipers, 2000, 2001; Mallot, Franz, Schölkopf, & Bülthoff, 1997; Mallot & Gillner, 2000; McNamara, Hardy, & Hirtle, 1989; O'Keefe & Nadel, 1978; Ruddle, Payne, & Jones, 1997; Schölkopf & Mallot, 1995).

To develop a complete understanding of human spatial navigation, it is important to understand how the cognitive map is developed and the nature of its representation; it is also important to understand the processes and strategies used in accessing and utilizing the information within the cognitive map. The current studies investigate issues associated with accessing information from the cognitive map. Specifically, these studies investigate how effectively subjects use available information in goal-directed navigation through familiar environments. In these studies, human performance is measured against that of a computer model, which makes optimal use of the available information. We use a measure termed efficiency, which is an index for comparing human performance to optimal performance. The computer model, implementing an optimizing principle, is in the tradition of “ideal-observer models” in perception. In our case, we also refer to it as an “ideal-navigator model.”

The findings from these studies provide us with important information about how effectively we use information in the cognitive map. In Experiment 1 we measured human navigation efficiency in environments that vary in their size. The primary finding is that human navigation efficiency declines as the environments become larger. The results from Experiment 1 demonstrate that there is a clear inefficiency in human way-
finding behavior. We hypothesize that this inefficiency may arise from one of four important processes involved in spatial navigation: perception; accessing the cognitive map; spatial updating or decision strategy. Experiment 2 investigated whether perception limited the subject’s ability to choose an efficient route through the environment. The findings suggest that perception was not the limiting factor. Experiment 3 investigated whether subjects had difficulty in accessing the cognitive map during navigation, or when updating their spatial position within the environment or in their decision strategy. By providing the subject with different types of information in different conditions, Experiment 3 found that most of the inefficiencies found in Experiment 1 appear to be due to inefficiencies in the subject’s spatial updating procedure.

It is important to appreciate that these studies investigated an observer’s navigation efficiencies; that is, how well did the participant move relative to an ideal observer with perfect perceptual processing, perfect map memory and the ideal decision strategy. By comparing human performance with that of the ideal observer, we are able to take into account variations in human performance that are due to task difficulty. This is because the ideal observer gives us the very best performance in the task. Because of this, the ideal observer is only sensitive to factors associated with task demands; any change in performance above and beyond these task demands can be attributed to the strategies and perceptual processing limitations of the human observer. In the following section we describe the ideal navigator used in these studies.

**Formalization of a Spatial Navigation Task**

This section describes the key properties in our model’s spatial navigation task. Later we build on some of these elaborations and formalizations to describe an ideal navigator that uses principles from Partially Observable Markov Decision Processes (POMDP) (Cassandra et al., 1994; Chung, 1960; Kaelbling et al., 1998; Sondik, 1971).

**Perception**

An observer navigating through a complex environment receives perceptual input in many different forms, including two-dimensional visual images (and possibly stereo depth if the observer has two eyes), auditory cues, vestibular input, kinesthetic feedback from joints and muscles, and processed forms of these perceptions that allow for path-integration. At any given moment during a navigation task one can specify all of this information in a high-dimensional perceptual vector \( P \). The perceptual vector is generated by specifying a specific state in the environment \( s_i \) and the observation function \( \square \) that converts the physical properties of the environment into a perceptual vector. The perceptual vector makes explicit all of the information that is available to the observer at any given moment during a navigation task.

\[
p_i = \square(s_i)
\]  

(1)

In addition to the perceptual vector, we can also specify a high-dimensional observation vector \( O \) that dictates which aspects of the perceptual vector are used during spatial navigation. This observational vector may store something as simple as a visual “snap shot” image from a particular state to something as complicated as an entire list of observable objects from a particular state (Gillner & Mallot, 1998; Mallot et al., 1997; Mallot & Gillner, 2000). One important aspect of comprehending human spatial navigation behavior is to understand what information is made explicit in memory given a specific perceptual vector. \( \square \) is the function that converts the perceptual vector into the stored observation vector given by Equation 2.

\[
o_i = \square(p_i)
\]  

(2)
Spatial Updating

Spatial updating is our ability to determine our location and heading in a large-scale space given our knowledge about the environment (cognitive map) and specific observations and actions while navigating. Spatial updating may rely on a number of perceptual inputs including proprioception, vision and audition. One form of spatial updating might simply include integrating the vestibular and proprioceptive cues (perhaps for path integration). In addition to these perceptual inputs, vision seems to play an integral role in our ability to update and move through an environment. The objects within the environment and the structure of the environment can give us important cues as to our current location and the set of actions that we need to take to reach our destination.

Decision Strategy

In spatial navigation the navigation system needs to interact with its environment. That is, a system has available to it a set of actions ($A$) that can move it from one state in the environment to another. An action might be as complex as “follow the center of a hallway until it ends”, or as simple as “rotate clockwise by 90°”. In defining a navigation task, one needs to specify the set of actions that are available to the navigator and how these actions are selected.

Navigation Goal

In order to formalize spatial navigation one needs to specify the goal. In different spatial navigation situations there may be different objective goals that the observer might pursue within the same environment. For example, the most common spatial navigation goal is to travel from one known location (like one's office) to another known location (perhaps the mailroom). Other times, the goal might be to figure out where you are located in the environment after getting lost. Another goal might be to determine which large-scale space you are located in out of a set of possible environments\(^1\). Each of these goals may produce different behaviors given identical perceptual input, actions, decision strategy and spatial updating procedures.

Ideal-Observer Modeling

An Ideal Observer Model provides optimal performance given the information available in the task. Typically ideal observers are not proposed as models of human cognition. Instead, the ideal observer provides a benchmark by which to compare human performance. More specifically, these models illustrate what optimal performance should look like. When human performance matches that of the ideal-observer model, one can conclude that the human is making use of all of the information in the task. When the human underperforms the ideal observer, specific discrepancies between the human data and the ideal data may illuminate the processing constraints imposed by the human cognitive system.

Ideal observer analysis has been used to understand cognitive functions from the quantum limits of light detection (Hecht, Shlaer, & Pirenne, 1942) to many forms of visual pattern detection and discrimination (Geisler, 1989), to reading (Legge, Hooven, Klitz, Mansfield, & Tjan, 2002; Legge, Klitz, & Tjan, 1997) and object recognition (Liu, Knill, & Kersten, 1995; Tjan, Braje, Legge, & Kersten, 1995; Tjan & Legge, 1998). In the current studies we are interested in understanding the cognitive limitations of human spatial navigation. To do this we developed an ideal navigator model. This model uses an optimal algorithm to solve a spatial navigation task. We can compare the performance of this model with the performance of human participants on the identical task. The model has available the same information that is available to the human participants.

---

\(^1\) This can happen when one exits an elevator on the wrong floor, or emerges from a train station at an unknown stop.
In the following section we formalize an ideal observer model for a specific spatial navigation task. The model uses principles from *Partially Observable Markov Decision Process* (POMDP) (Cassandra et al., 1994; Kaelbling et al., 1998) to navigate through familiar indoor environments. The goal of the model is to travel from an unknown starting point to a known target location using, on average, the fewest number of actions. We use the model to measure the expected change in performance due to task demands.

**Ideal Navigator**

We will describe the ideal navigator in two sections. The first will provide an intuitive appreciation of what the navigator is doing, and the second will provide a more formal description of the model.

**Intuitive Explanation**

The model has perfect knowledge of the environment. This knowledge comes in two forms. First, the model knows what it expects to see from every state within the environment. Second, the model knows the connection matrix for the entire environment. In other words, the model knows exactly where it will end up if it executes a particular action at a particular place and orientation in the environment. Given that it knows what it expects to see from every state in the environment, it will also be able to determine what it expects to see in the new state following the action.

In the current rendition of the model, the model starts from an unknown location (i.e., placed at a random location within the environment) and is instructed to move to a known target location in as few actions as possible. Here are the steps that the model takes to choose the next action.

1. **State Elimination**: Compare the current view with the views that the observer would expect to see in the set of potential states\(^2\). Eliminate the states from the set of potential states that are not consistent with the current view.

2. **Route Generation**: From each of the potential states compute the shortest route that starts with each action from the set of available actions (i.e., rotate-left, rotate-right, and forward). The route has to reach the goal state in the fewest moves with no remaining uncertainty.

3. **Action Cost**: Step 2 provides a list of routes from each remaining state that starts with a particular action (in the current rendition of the model there can be up to 3 routes for each state — one for each action). For each action, compute the average number of moves that it would take to reach the goal state from each of the states. This will provide up to three averages (one for each action).

4. **Action Selection**: Choose the action that has the minimum number of moves on average (from step 3).

5. **Potential State Updating**: Update the set of potential states by computing what state the observer would be in if they executed the selected action from Step 4.

6. **Action Execution**: Execute the action in the real environment.

7. **Check if Done**: Check if at target location with no remaining uncertainty.

   *Yes*: done
   
   *No*: Return to step 1.

---

\(^2\) Initially the set of potential states are all of the states in the environment. However, after the initial view and set of actions, the model eliminates certain states from consideration. Those that remain are the “potential states”.

5
This algorithm provides a set of actions that minimizes the set of moves on average to reach the goal state. There are some properties of this algorithm that we would like to point out. First, the algorithm minimizes the set of actions on average to reach the goal state. Given that there is uncertainty about the observer’s position in the environment, the model does not always take the direct route from the starting state to the target state. Typically, this occurs when the best move from most of the states is one move (e.g., rotate-left), but the best move from one of the states is different (e.g., move forward). Given the uncertainty, the best move is “rotate-left”. However, if the observer is starting from this aberrant state, the model may need to “backtrack” after a few moves.

**Formal Description**

The model has perfect knowledge of the environment. This knowledge comes in the form of knowing the observation vector for each state in the environment (see, Equation 1). Furthermore, the model can invert the function such that it can produce the state or set of states that are consistent with a particular view. It should be noted that for any given state in the environment there is a single observation vector, but for a particular observation vector (o), there may be more than one state that could have produced that vector.

Given a single observation there may be some amount of state uncertainty (i.e., more than one location may generate the same observation). We can represent this uncertainty by specifying a belief vector (B) that indicates the probability that the observer assigns to being in each of the states of the environment (S). We can write b(s) as the probability assigned to state s when the observer’s belief state is b.

In addition to the belief state vector, and the observation vector, there is also an action vector A. The action vector is the set of movements or actions the observer can make within an environment. Given a belief vector, observation vector, and the action vector, the model requires a function for estimating the observer's state within the environment. This state estimator takes as input the current belief vector, the previous action and the current observation and returns an updated belief state. To update the current belief state one simply needs to apply Bayes' rule for estimating the likelihood that the observer is in a given state s:

\[
SE_s = p(s' | a,o,b) \\
SE_s = \frac{p(o | s',a,b)p(s' | a,b)}{p(o | a,b)}
\]  

(3)

**Optimal Move Decisions**

The goal of the system is to reach a belief state in which the observer is at the target location with no uncertainty (b(T)) using the fewest number of actions (i.e., he lowest cost). In the model, each move has an associated cost (C). For each state (s) we can compute a set of routes (R) that will move the observer from a state (s) to the goal state (T) with no remaining uncertainty. For each state there are multiple routes to the target state (T). For each of these routes we can compute a cost associated for that route (Cost(r,s,b)) as specified in Equation 4.

\[
Cost(r,s,b) = b(s) \bigwedge_{a \in r} C_a
\]  

(4)

---

3 There is no random noise in the perceptual or observation vector in the current version of the model. The uncertainty occurs because multiple states may generate the same observation vector.

4 In the current studies the ideal observer and the human observer have three actions available to them: rotate-right 90°, rotate-left 90°, and move forward one hallway unit.
r is the vector of actions that can move the observer from state s to the target state. Note that the cost is multiplied by the likelihood that the observer is located at a particular state in the environment.

Because the model may be in a state of uncertainty, the model only plans an optimal move one move ahead. The reason for this is that after the move, the observer will collect new perceptual information that will modify the belief vector (see Equation 2). For each state s the model computes the least expensive route that starts with each action \( r_i = a \):

\[
StateActionCost(s, a \in A) = \min \left[ Cost(r_i = a, s, b) \right]
\]  

(5)

After computing the cost of making each action from each state, the model then selects the action that minimizes the cost across all of the states in the environment:

\[
Move = \min_{a \in A} \left[ \min_{s \in S} StateActionCost(s, a) \right]
\]  

(6)

After choosing the action selected by Equation 6 and making the action, the model receives a new observation vector \( o \). The model then updates its belief vector using the new observation vector \( o \) and the selected action \( a \) using Equation 3.

**Experiment 1: Effect of Layout Size on Spatial Navigation Performance**

In Experiment 1 we were interested in understanding the effect of increasing layout size on human spatial navigation performance. The purpose of this study was two-fold. First, we wanted to determine if subjects were inefficient at navigating with state uncertainty, and second, we wanted to investigate how these inefficiencies change as a function of the layout size. Increasing the layout size should increase the demands placed on an observer’s perceptual processing (i.e., short hallways versus longer hallways), accessing the cognitive map, belief vector updating and decision strategy. We hypothesized that if subjects have a cognitive limitation in one of these functions it should become evident as a decline in efficiency as the environment becomes larger. To investigate this issue we used four different layout sizes defined by the number of hallway segments in a layout. The four levels of layout size were: 10, 20, 40, and 80 hallway units\(^5\) (See Figures 1, 2 and 3).

In this experiment, we trained subjects to navigate through these virtual environments until they attained a specified learning criterion. Once proficiency was established, we then started subjects from a random state within the environment and instructed them to move to a target location using the fewest possible actions and be certain that they are at that location. Because the environments were perceptually sparse (no object landmarks), the subject could start a test session with considerable state uncertainty. We also ran the ideal navigator through the equivalent environments with the same set of actions and the same goal.

**Methods**

**Apparatus.** The experiment was run on an IBM Dell computer with a 19” color monitor.

\(^5\) A single hallway unit is a 30-foot long hallway that was used to generate the random environments used in the current study. These hallway units were randomly placed on a grid to generate environments similar to the map shown in Figure 1.
The subject moved through the environment by making key presses that corresponded to a 90° clockwise rotation, a 90° counter-clockwise rotation or a forward translation of one hallway unit.

After the subject made a key press, the computer would rotate or translate the virtual "camera" in the virtual space. The camera would produce the appropriate optic flow for the action indicated by the key press. A rotation was completed in 750 milliseconds and a translation was completed in 900 milliseconds.

**Stimuli.** For each layout size (10, 20, 40, 80 corridors) two different environments were produced for a total of eight different environments. Figure 1 provides an illustration of an environment of size 20. Each layout had one common feature. They all had an L-junction on the exterior part of the layout (see lower left hand corner of Figure 1). The end of this L-junction served as the starting point for each trial of the exploration phase of the training session.

![Figure 1](image-url) Figure 1. An example of the types of maps generated by the random layout generator. This environment is a 20-hallway environment. Each line in the figure represents a hallway. The end of the lower left hand L-junction served as the starting point for each trial of the Exploration sessions. Figure 2 illustrates an example of a first person rendering of an environment used in Experiment 1.

The environments were randomly generated. We generated the environments by specifying three parameters: the number of hallway units comprising the environment, the maximum number of vertical hallway units and the maximum number of horizontal hallway units that the environment could have (plat size). The plat size for the current studies was 20 hallways by 20 hallways.

To generate an environment the computer began by randomly selecting one of the 400 potential hallways in the plat. After selecting a hallway, the computer then computed the set of potential hallways. Potential hallways were all of the hallways that connected to the current set of selected hallways. The computer then selected one of the potential hallways and removed the hallway from the list of potential hallways. It then re-computed the new set of potential hallways. The computer continued with this process until it reached the layout size minus 2 hallways. At the end, two hallways were added to the environment (an L-junction) that served as the starting point during the exploration phase of the experiment.
The environments were rendered from a first-person perspective with the eye height of the camera (in the virtual environment) placed at 5 feet. Figure 2 provides a sample view of the environment from the subject's perspective. To increase the subject's ability to differentiate between an intersecting hallway and a wall, red railings were placed at junctions on walls where there was no intersecting hallway (see Figure 2 on the left).

To ensure that all of the hallways were identical, we used an “environment parts kit” to generate the environments. This parts kit consisted of three basic parts: a hallway, an intersection node and a wall. To generate a virtual environment these parts were placed in specific configurations to give the appropriate environment layout. Figure 3 illustrates the parameters and properties of the parts kit. Table 1 lists details about the parameters of each part including the rendering color and/or texture.
Table 1. Properties of the environment parts kit used to generate the random environments for Experiments 1 and 2

<table>
<thead>
<tr>
<th>Item</th>
<th>Height (ft.)</th>
<th>Width (ft.)</th>
<th>Length (ft.)</th>
<th>Color/Texture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hall Floor</td>
<td>--</td>
<td>10</td>
<td>30</td>
<td>Burlap</td>
</tr>
<tr>
<td>Hall Ceiling</td>
<td>--</td>
<td>10</td>
<td>30</td>
<td>Cement</td>
</tr>
<tr>
<td>Hall Wall</td>
<td>10</td>
<td>--</td>
<td>30</td>
<td>Cement</td>
</tr>
<tr>
<td>Hall Railing</td>
<td>1</td>
<td>0.5</td>
<td>30</td>
<td>White</td>
</tr>
<tr>
<td>Hall Light</td>
<td>0.5</td>
<td>8</td>
<td>28</td>
<td>Purple</td>
</tr>
<tr>
<td>Wall</td>
<td>10</td>
<td>--</td>
<td>10</td>
<td>Cement</td>
</tr>
<tr>
<td>Wall Railing</td>
<td>1</td>
<td>0.5</td>
<td>4</td>
<td>Red</td>
</tr>
<tr>
<td>Node Floor</td>
<td>--</td>
<td>10</td>
<td>10</td>
<td>Burlap</td>
</tr>
<tr>
<td>Node Ceiling</td>
<td>--</td>
<td>10</td>
<td>10</td>
<td>Cement</td>
</tr>
<tr>
<td>Node Light</td>
<td>0.5</td>
<td>6</td>
<td>6</td>
<td>White</td>
</tr>
</tbody>
</table>
Figure 3. To generate the environments we used an “environment parts kit” that consisted of a hallway, a node, and a wall. All of the structures were generated using these three fundamental parts. Some of the measurements are given in the graphic here; more of the details are listed in Table 1. The upper graphic shows the measurements when looking at each part from above and the lower graphic illustrates the layout when looking at the environment from the side.

Procedure. Subjects participated in a training session and in a test session for each condition of the experiment. The training session was designed to provide the subject with a detailed representation of the environment. The training session consisted of two phases: an exploratory phase and a drawing phase. In the exploratory phase the subject started from an external L-junction. The subject explored the environment for three minutes by making key presses on the number pad to indicate the movement that he to make. The “8” corresponded to a forward movement, while the “4” and “6” corresponding to rotate counter-clockwise and clockwise rotations respectively. During the training phase, subjects learned both the layout of the environment and a target location. The target location was specified by an auditory signal (the sound of a bell) each time the subject walked over the target location. Subjects were told that later in the experiment they would start from a random place in the environment and would need to move to the target location making as few actions as possible.

After exploring the environment for three minutes, the subject participated in the drawing phase. In the drawing phase, the subject was given a grid pattern that had a single L-junction placed near the center of the grid that corresponded to the starting location in the exploration phase of the study. The subject was told to “connect the dots” to recreate the environment he had just explored. Subjects were informed that each dot could be thought of as a node, or the stopping location when they made a forward movement through the environment\(^6\). If the subject’s map drawing did not perfectly reproduce the grid layout of the environment, then

\(^6\)To be certain that subjects understood this process, they were first trained on a small (10 hallway) layout environment before starting the experiment.
the subject participated in another three-minute exploration session followed by another drawing test. Subjects continued in the three-minute exploration phase followed by the drawing phase until they drew the environment correctly twice in a row.

After reaching criterion in the training phase, the subjects entered the test phase of the experiment. In the test phase, subjects started from a random state (i.e., a random location and orientation) in the environment. Subjects were instructed to move to the target location using as few actions (key presses) as possible. They were informed that a rotation and a translation were both considered an action. When subjects reached the target location there was no auditory signal indicating they were there. Instead, subjects were required to indicate when they believed they had reached the target location by pressing the spacebar on the computer. After pressing the spacebar the screen went white, when the subject was ready to begin the next trial, the subject pressed the spacebar a second time to reveal their new starting view.

Subjects started from each position in the environment an equal number of times; the computer randomly selected a starting orientation from the four that were possible. Each subject participated in 320 trials in each of the environments. Subjects participated in each of the eight environments in a random order.

Subjects. Three female subjects ran in the experiment. Each subject had normal or corrected to normal vision. The ages of the subjects ranged from 21 to 24 years of age. The subjects were paid $8.00 per hour to participate in the study.

Results

The primary dependent measure in Experiment 1 was the number of moves that it took subjects to reach the target location. In order for subjects to solve this task, they had to determine both their location within the environment (because they are starting from a random location) and the shortest path (fewest actions) to the target location. The upper graph of Figure 4 illustrates the average number of moves subjects took as a function of layout size. Trials in which subjects indicated the target location at the wrong position, or trials in which it would have been impossible for the human to have reached the goal state with no remaining uncertainty were excluded from the analysis (combined, these trials were less than 1% of the trials). Figure 4 also illustrates the number of moves, averaged across layouts of a given size for the ideal navigator\(^7\). The number of actions for the ideal observer increases initially (from 10-20 hallways) with the effect of layout size having a diminishing effect between 20 and 80 hallways.

The lower plot of Figure 4 illustrates an “efficiency” measure that allows us to factor out task difficulty in human behavior. Efficiency is simply measured as the number of actions required by the ideal observer divided by the number of moves made by the human. The lower plot of Figure 4 illustrates the efficiency functions for the three subjects. A one-way ANOVA revealed a significant effect of layout size on movement efficiency for each of the subjects (JAM F(3)=5.422, \(p<0.01\); JUL F(3)=32.11, \(p<0.01\); SMS F(3)=22.03, \(p<0.01\)). Human efficiency dropped from approximately 80% for the smallest environment (10 hallway units) to approximately 50% in the largest environment (80 hallway unit environments).

\(^7\)The ideal observer performance was computed for each subject in each environment. That is, the model started from the same states as the human observers. Because there was no significant variation across the subjects, we plotted the average performance across the trials, subjects and environments.
Discussion

The results from Experiment 1 show that subjects’ navigation performances are sub-optimal. This is demonstrated by the fact that the efficiency is below 1.0 across each of the layout sizes. The results also show
that subjects become less efficient as the size of the environment becomes larger. The sources of these inefficiencies can have multiple origins which can be broken down into four primary categories: perception; accessing the cognitive map; spatial updating and decision strategy.

**Source of the Cognitive Limitation**

Using the ideal navigator we can begin to investigate where the cognitive limitations might exist in spatial navigation. One model that we will consider is Schölkopf & Mallot’s (1995) view-graph model of spatial navigation. This model makes associations between specific actions and views in order to reach the goal location.

**Limited Perceptual Processing**

One function that might be limiting human navigation performance is the visual information used by the human observer. Although the environments used in Experiment 1 were very sparse, the observers may not have processed all of the hallway information while navigating. For example, subjects may simply have identified the structural information up to the next hallway unit, and then ignored the structural information beyond that intersection. Of course this perceptual limitation could be extended so that subjects might only process \(N\) closest hallways and ignore the perceptual information beyond that. A perceptual limitation like this would affect navigation in large environments more than in smaller environments because the average length of a corridor in the larger environments is longer than in the smaller environments.

**Accessing the Cognitive Map**

Subjects may have difficulty accessing the entire cognitive map while they are navigating. Although subjects are required to draw the environment correctly twice in a row before they can participate in the testing phase, they might not be able to access the entire cognitive map with sufficient precision to make the relevant comparisons implicit in ideal performance. Facility in accessing the global cognitive map may also be adversely affected by the concurrent task of navigating through the environment. Therefore, it is plausible that working memory capacity is preventing subjects from loading the entire cognitive map while they are navigating. If subjects have a limited working memory capacity that only allows them to load a limited amount of the layout information, then as the environment becomes larger, the proportion of the entire layout that is being considered drops with a corresponding reduction in navigation efficiency.

**Spatial Updating**

A third cognitive limitation may arise from the subjects’ ability to update their position within the environment. In the current experiment the subject starts from an unspecified location within the environment. When subjects make their first observation, there may be many locations consistent with this observation; in other words, there may be substantial ambiguity. We quantify this ambiguity with the belief vector. As the subjects make actions within the environment and obtain new observations, the belief vector should change and the uncertainty should decrease until ultimately they know exactly where they are located. One possible source of sub-optimal human performance might be due to the subject’s inability to accurately update this belief vector while moving through the environment. Subjects may be limited in the number of locations they can consider (e.g., 7 +/- 2 places; (Miller, 1956)) or they may not possess an optimal updating strategy. Because the larger environments consist of more locations, the number of states in which the observer could be in while navigating
also increases. Thus, if subjects have a sub-optimal spatial updating strategy or limited belief vector memory, their performance will be negatively affected as the environment becomes larger.

**Sub-Optimal Decision Strategy**

Finally, the strategy used by the subjects may be sub-optimal. For example, one sub-optimal strategy is an inappropriate cost function placed on each action. The subjects in the experiment were explicitly instructed to make as few actions as possible to reach the goal state. This would place an equal cost on translations and rotations. It is possible that, despite these instructions, subjects did not treat each action as having an equivalent cost. For instance, they may have associated a larger cost with a translation than a rotation. Another example of a sub-optimal decision strategy would be if subjects first localized themselves within the environment and then moved to the target location.

**View-Graph Strategy**

One model that does not use ideal decision making strategy is the View-Graph model of spatial navigation proposed by Schölkopf & Mallot (Schölkopf & Mallot, 1995). In this model, spatial navigation is accomplished by storing a series of views in memory and for each view, assigning a specific action for reaching a particular destination. This model has been used in robotics for learning and navigating through complex environments. Recent research by Gillner and Mallot (1998) found evidence that human subjects may use this type of strategy while navigating through complex environments. It is possible that subjects were engaging in this type of sub-optimal decision strategy to accomplish this task. To test whether subjects are engaging in this strategy we investigated how well one can predict the human action given a particular view. If subjects are using this strategy for navigating through these environments, we would expect to be able to predict their selected action for each view. To investigate this question we measured the conditional entropy between the action selected (A) and each view in the environment (see Equation 7). According to the view-graph model of spatial navigation humans should choose the same action (A) given a specific view. That is, the View-Action entropy should be relatively low (or possibly 0).

\[
H(A|V) = \sum_{v \in V} p(v) \sum_{a \in A} p(a|v) \log \frac{1}{p(a|v)}
\]  

(7)

As a comparison, we also computed the conditional entropy for the ideal navigator. The navigator is not in any way constrained by a view-action association. The model instead selects the optimal action given its state of uncertainty and knowledge of the environment. The model serves as a baseline by which to compare the human data. It provides a lower bound on the action entropy if a system is making optimal use of the perceptual information and is not limited in its strategy.

Figure 5 shows that the ideal navigator’s conditional entropy is lower than that for each of the subjects. That is, it is more difficult to predict the human action (given the view) than the ideal navigator’s action. This finding suggests that if subjects are limited by their decision strategy in Experiment 1, they do not seem to be limited in the way suggested by the View-Graph model.
Summary

Experiment 1 investigated the effect of increasing layout size on human navigation performance. We measured the number of actions a subject used to move from an unspecified location in a familiar virtual reality environment to a target location. Four layout sizes were used: 10, 20, 40 and 80 hallways. The raw data showed that subjects required more moves to reach the goal as the layout size increased. This finding could be due to a combination of two factors: cognitive limitations and/or task demands. To distinguish between these alternatives, we developed an ideal navigator with no perceptual, memory or strategy limitations (cognitive limitations). The model’s performance was compared with human performance. We found that the increase in the number of human actions out-paced that of the ideal navigator, suggesting there is a cognitive limitation that is affecting human spatial navigation.

We hypothesize that the inefficiency found in this experiment might be due to inefficiencies in perceptual processing, accessing the cognitive map, spatial updating or decision strategy. In Experiment 2 we investigated whether the inefficiency can be explained by an inefficient perceptual processing system. In Experiment 3 we investigated whether the inefficiency is in accessing the cognitive map, spatial updating or the decision strategy.
Experiment 2: The Effect of Limited Visual Information on Spatial Navigation Performance

Experiment 2 examined whether human navigation performance is limited by visual-information processing constraints. For this experiment, the subjects performed the same navigation task as in Experiment 1. However, rather than manipulating the size of the layout, we manipulated how far the observer could see along each corridor (the View Depth). This was accomplished by adding “virtual fog” to the environment (see Figure 7). As in Experiment 1, we compared the navigation performance of human observers to the performance of the ideal navigator to obtain navigation efficiency. This allowed us to factor out task-difficulty to reveal the cognitive limitations in human navigation.

If human navigation performance is limited by visual-information processing restrictions when the view-depth is unlimited (i.e., subjects can see to the end of a long hallway), then, humans ought to be unable to use all the visual information in the visual display. The ideal navigator does not have such a limitation and will use all the available information in making its navigation decisions. Thus, when the view depth is limited (by adding virtual fog), it should have a larger impact on the ideal observer than on the human observer. Accordingly, we expect to see an improvement in efficiency when the View-Depth is limited.

Methods

Apparatus. The apparatus was the same as in Experiment 1.

Stimuli. View depth (how far the observer could see down a hallway) was manipulated by adding virtual fog to the environments to limit how far down the hallway an observer could see. Figure 7 illustrates this effect by showing a view without fog (unlimited viewing condition) and a view with fog (fog depth = 1 hallway). The fog provided a method for manipulating viewing depth that felt relatively natural.

Figure 6 provides an analysis of the view information available for the four environments used in Experiment 2. One concern in running this study was whether the fog manipulation in fact limited the perceptual information available to the subject (i.e., all of the hallway lengths could be less than 1). Figure 6 plots the frequency of states as a function of the number of collinear hallways for each environment (i.e., hallway length). In the current studies, the Limited-View condition would limit the perceptual information available in the states where the hallway length is greater than one. Across the four environments this constitutes 36% of the states.
Figure 6. View-depth analysis of the 4 environments used in Experiment 2. The graph shows the number of states (Frequency) that have a specific number of collinear hallway units that can be observed from that state. For example, when the subject is looking at a wall, the number of collinear hallway units is 0. Observers in the Unlimited-View condition would be able to see all of the hallway units. Observers in the Limited-View condition would be able to see all of the information in the 0-view-depth states, but in the states with a view depth greater than 1 they would be limited in the visual information that they could obtain.

Procedure. Four different environments were used in Experiment 2. Each environment consisted of 40 hallway units. Subjects were tested in all 4 environments: two with limited view depth and two with unlimited view depth. Each environment was used in all of the conditions across the subjects.

The training procedure was the same as in Experiment 1. When subjects trained for the limited view condition, they completed the training session with limited visual input.
Figure 7. Sample views from Experiment 2. The image on the left illustrates the view in the unlimited view condition, and the image on the right illustrates the same perspective in the limited view condition. Note that in the image on the left the subject can determine the hallway structure for the next node, but beyond that node the subject has no perceptual information.

Subjects. Three subjects were used in Experiment 2: two females and one male. Their ages ranged from 20 to 22 years old. All subjects had normal or corrected-to-normal vision.

Results

Figure 8 plots the number of moves made by the human with limited and unlimited visual information. The plot on the right of Figure 8 illustrates the subjects’ performance relative to the ideal navigator in the same environment with identical visual information. The primary question of interest is whether humans become more efficient when view depth decreased. For the three subjects there was no systematic change in efficiency in the two conditions. The change in performance was relatively small (+/- 4%) and went in both directions across the three subjects.

Table 2 Analysis of Variance (ANOVA) for number of moves and efficiency measures for each subject in Experiment 2

<table>
<thead>
<tr>
<th>Subject</th>
<th>Actions</th>
<th>Action Ratio</th>
<th>Efficiency</th>
<th>Efficiency Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub3</td>
<td>F(1)=3.03, p=0.0823</td>
<td>1.0955</td>
<td>F(1)=20.89, p&lt;0.05</td>
<td>0.7632</td>
</tr>
<tr>
<td>Sub4</td>
<td>F(1)=5.28, p&lt;0.05</td>
<td>1.1373</td>
<td>F(1)=2.19, p=0.1393</td>
<td>0.9128</td>
</tr>
<tr>
<td>Sub5</td>
<td>F(1)=3.19, p=0.0747</td>
<td>0.9072</td>
<td>F(1)=7.38, p&lt;0.05</td>
<td>1.1974</td>
</tr>
</tbody>
</table>
Figure 8. The effect of reducing visual information on human navigation performance in Experiment 2. The graph on the left shows the average number of moves for the three subjects when navigating with unlimited and limited visual information. The graph on the right shows the subjects' performances relative to the ideal navigator.

Although there was a significant increase in efficiency for one subject, there was a significant decrease for another, and the third subject showed no significant change in performance. Overall, the effect sizes were relatively small.

**Discussion**

This experiment investigated the effect of limiting visual input on spatial navigation performance. The purpose of this study was to determine whether the cognitive limitation found in Experiment 1 was due to limited visual processing. For example, in Experiment 1, subjects might have processed information only one or two hallway units down a corridor view. If subjects' performances were limited by visual processing, then the subjects should be less efficient in environments that have more long corridors than those with fewer long corridors. The smaller environments in Experiment 1 have fewer long corridors than the large environments.

The findings of Experiment 2 suggest that limited visual processing cannot explain the efficiency reduction found in Experiment 1. Although we ran subjects who had normal vision, one can apply the findings from these studies to low vision navigation. Low vision refers to people who have uncorrectable visual impairment. The current studies used environments that have sparse visual cues and in Experiment 2, we further limited how much visual information was available to the observer. This may simulate certain aspects of low vision navigation. Previous research investigating the effect of simulated low vision on obstacle avoidance performance found effects similar to that in Experiment 2. Pelli (1987) investigated the effects of simulated reductions of acuity, contrast and visual field loss on a subject’s ability to avoid obstacles. He concluded that subjects had to have extreme limitations in acuity, contrast or visual field before there was a decline in obstacle avoidance performance. The results from Experiment 2 suggest that moderate limitations in visual information may also have only a small effect on way finding behavior.

---

8 A corridor is a series of co-linear hallways
Experiment 3

The current experiment investigates whether the inefficiency found in Experiment 1 is due to inefficiencies in: a) accessing the cognitive map, b) spatial updating or c) decision strategy. In Experiment 3 we did not manipulate the layout size (we used only one layout size: 40 hallways) but instead provided subjects with supplementary map information while they navigated to the target location from an unspecified starting location. The experiment had three conditions: No-Map, Map, and Map+Belief Vector conditions. The No-Map condition allowed us to measure baseline efficiency performance for the task. In this condition the subjects simply saw a blank square on the lower left corner of the screen while they were navigating through the environment (see Figure 9).

In the Map condition of Experiment 3 we provided a map of the environment on the computer monitor while the subject was navigating. Because Experiment 3 used the same map drawing technique as in Experiments 1 and 2, we know that subjects’ cognitive representations are sufficient to generate a global map of the environment. But it is possible that generating the map is computationally burdensome, and that during navigation it is not easy to maintain access to a global cognitive representation of this type. By using a map display the subjects were not required to access a cognitive map during navigation. Instead they could reference the visual map presented on the display. Thus, we hypothesized that if human inefficiency is due to problems accessing the cognitive map, then we should find a significant increase in performance from the No Map condition to the Map condition.

Alternatively, subjects might be inefficient in their spatial updating ability. Remember that in this experiment subjects are starting from an unspecified location in which the initial view can leave the observer with state uncertainty. As subjects move through the environment they have to use the perceptual information and the action selections to update where they believe they are in the environment. Or in terms of the model, they have to update their belief vector. In the Map + Belief Vector condition we presented the observers with the map and the target location. Superimposed over the map was an accurate belief vector, assuming that all of the perceptual information was being used (see lower left and right panels of Figure 9). This belief vector was updated after every action made by the observer. The computer updated the current belief vector by factoring in the current and previous observations and the previous actions. We hypothesized that if the navigation inefficiencies found in Experiment 1 were due to spatial updating, adding the belief vector information should show an increase in movement efficiencies from the Map condition to the Map + Belief Vector condition.

Finally, subjects might simply have an inefficient movement strategy. It is possible that subjects might be accessing the cognitive map perfectly, and also generating an accurate belief vector, but their action selection strategy is inefficient. For example, subjects may decide to choose the action that minimizes the overall distance between their current position and the goal state. Although under some conditions this might be a good strategy, in others it might lead subjects down dead-end hallways, which would be inefficient. If this is true, then adding the supplemental information will not increase the subject’s efficiencies at all. Thus, if the inefficiencies are in the action selection strategy, then we predict there would be no difference in performance across the three conditions.

Methods

Apparatus. The apparatus was the same as in Experiment 1.

Stimuli. Experiment 3 manipulated the supplemental map information available to the subject. There were three types of supplemental information: No Map, Map, and Map + Belief Vector. Figure 9 provides a sample illustration of how these conditions appeared to the subject.
In each condition there was a black square that was presented in the lower left quadrant of the display. The location of this display was carefully chosen so that it would not block any of the informative information during the experiment. In the No Map condition this square was blank; in the Map condition a static map image was superimposed over the black square. In addition to the map, there was a blue square indicating where the target location was in the environment. In the Map + Belief vector there was the map plus the target location symbol superimposed on the black square. In addition to this, there was a collection of red pointers indicating where the subject could be located in the environment given the subject’s current view, previous views and actions. These pointers were updated after each of the subject’s actions\(^9\).

Procedure. Experiment 3 used one environment that consisted of 40 hallway units. The training procedure was the same as in Experiment 1. After reaching criterion (i.e., drawing the environment correctly twice in a row) subjects then started the testing phase.

During the testing phase subjects started from each state in the environment three times: once for each of the three conditions. The order of state+supplemental information conditions was run in a randomized order for each of the subjects.

Subjects. Four subjects were used in Experiment 3. Two females and two male ranging in ages from 20 to 22 years old. All subjects had normal or corrected-to-normal vision. Subjects 6, 7, and 9 were all volunteers who worked in the lab. Subject 8 was a paid subject who received $10.00 / hour for his participation in the study.

---

\(^9\) In the current studies there was no noise in the actions nor was their any uncertainty in the observation vector given to the model. Because of this, the probability that a subject could be in a particular state was either 0, or 1/N (where N is the number of potential states in the belief vector).
Figure 9. Illustration of the three conditions as viewed by the observer. The upper right illustration shows a view when there is no map information provided. The upper right panel shows a view when the map is provided. In addition to the map, the location of the target position was also shown by a small square at that position. The lower right panel shows a view when the observer sees the map with an accurate belief vector superimposed on top of it. The lower left panel shows an enlarged version of the belief vector map. The arrows on the map show where the subject could be given the previous views and actions along with the current view. Note that not all of the dead-ends are shown as possible locations. This is because this image was generated after a sequence of actions.
Results

Experiment 3 manipulated the supplemental information available to the subject while navigating from an unspecified location to a target location. For each subject we computed their Movement Efficiency by taking the ratio of the number of moves required by the ideal observer relative to the number of moves required by the human observer.

Figure 10 illustrates these data for the three supplementary map conditions. Planned comparison T-Tests found that there was no significant difference between the No Map and the Map conditions for all four subjects (see Table 3) but there was a significant effect between the Map and the Map+Belief Vector condition for all four subjects.

Figure 10. Action selection efficiency plotted for the four subjects in Experiment 3 as a function of the supplementary information provided to the subject while navigating.
Table 3. Planned, paired t-tests for the four subjects in Experiment 3.

<table>
<thead>
<tr>
<th>Subject</th>
<th>No Map vs. Map</th>
<th>Map vs. Map + Belief Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub6</td>
<td>T(126)=-1.881; p=0.062</td>
<td>t(127)=-5.791; p&lt;.001</td>
</tr>
<tr>
<td>Sub7</td>
<td>t(127)=.142; p=0.887</td>
<td>t(127)=-6.764; p&lt;.001</td>
</tr>
<tr>
<td>Sub8</td>
<td>t(124)=-0.769; p=0.443</td>
<td>t(124)=-6.796; p&lt;.001</td>
</tr>
<tr>
<td>Sub9</td>
<td>t(125)=0.463; p=0.644</td>
<td>t(127)=-5.848; p&lt;.001</td>
</tr>
</tbody>
</table>

**Discussion**

The findings from Experiment 3 are very clear. There was no significant increase in performance from the No Map to the Map condition. This result suggests that subjects did not have difficulty in accessing their cognitive map, or that the global information afforded by the supplementary map was not useful to them. By contrast, there was a significant improvement in performance from the Map to the Map + Belief Vector condition (from approximately 60% efficiency to 95% efficiency). This large increase suggests that most of the inefficiency in Experiments 1 and 2 is due to the processes involved in the subject’s ability to update his/her belief vector. The belief vector is a list of states that the observer (human or ideal) could be in given their previous observations and actions. After the subject makes an action and makes a new observation within the environment he/she will update his/her belief vector (i.e., update the set of states the observer believes that they could be in). It seems that for some reason the subjects are having difficulty integrating the set of observations and actions with their cognitive map to generate an accurate list of states.

The results from the current studies provide important information about sources of human inefficiency while navigating, but they do not tell us why humans are inefficient. Understanding why subjects are inefficient at updating their belief vector will be addressed in future research. Some sources for the inefficiencies in updating the belief vector may be memory or updating strategies. These and other questions will be investigated in the future.

**General Discussion**

Spatial navigation is composed of multiple processes that include perception, memory, spatial updating and decision-making. A breakdown in any one of these processes can have detrimental affects on human spatial navigation performance. The series of studies described here investigate navigation efficiencies as a function of the size of the environment, with reduced visual input, and with supplemental layout information. The goal of these studies was to understand how efficient human spatial navigation was under conditions of state uncertainty (i.e., when one does not know one’s exact position and orientation within a familiar environment). Each of these studies compared human performance to that of the ideal navigator, based upon principles from Partially Observable Markov Decision Processes (Cassandra et al., 1994; Chung, 1960; Kaelbling et al., 1998). The findings from the current study provide insight in some of the cognitive limitations in navigating through indoor environments. First, Experiment 1 shows that subjects become less efficient at their ability to navigate through large-scale spaces as the environments become larger (in terms of hallway units). This suggests that there is some sort of cognitive limitation that is correlated with layout size.

One possible limitation might be in the analysis of the visual information that is available from each view. Experiment 2 was designed to address this issue. By limiting the visual information for the human and ideal observer we were able to see whether human efficiency changed under these two conditions. There was no change in efficiency, which suggests the results from Experiment 1 were not due to a limitation in processing perceptual information.
Experiment 3 was designed to address whether the limitation might be due to accessing the cognitive map from memory, considering multiple states simultaneously or the subject’s decision strategy. The results from this study clearly showed that a large part of the limitation is due to ineffective use of using preceding views and actions in resolving.

**Navigating with Uncertainty**

The current studies suggest that subjects have difficulty navigating when there is state uncertainty. From the current studies we are not able to isolate the source of this uncertainty, but there seem to be several of possible sources. One such source may be a simple memory limitation. In one of our experiment’s most extreme case in one of our experiments the subject could have to contend with 72-fold state ambiguity (i.e., when the subject is facing a wall in the layout of size 80 in Experiment 1) in order to efficiently navigate through the environment. This far exceeds the typical estimation of human short term memory of seven +/- 2 items (Miller, 1956). When there is a high degree of state uncertainty in navigation, humans may only consider a subset of the possible states. Under this condition they may navigate until they receive perceptual input that is inconsistent with all of the remaining possible positions; they would then have to regenerate a new belief vector.

**Spatial Navigation and Low Vision**

The current studies also provide some insights into low-vision navigation. Low vision is any uncorrected visual disorder that impacts an individual’s daily life. Glaucoma, cataracts and macular degeneration are some of the disorders that lead to low vision.

For individuals with low vision, spatial navigation can be a very daunting task. Many visual cues that are used by people with normal vision are unperceivable by someone with low vision (e.g., door signs, small object landmarks such as phones, or drinking fountains). In the current experiments, we had normally-sighted subjects navigate through environments that were visually sparse. That is, there were no object landmarks in the environment to help with localization. Instead, subjects had to deduce their position in the environment by stringing together a series of views and actions. This was especially true in the limited-view condition in Experiment 2. The results from Experiment 1 suggest that under sparse viewing conditions that produce a large amount of perceptual aliasing (more than one state producing the same perceptual image), subjects are negatively affected by increasing the size of the environment. However, as Experiment 3 showed, when subjects were given an explicit reference to the set of states where they might be, performance improved dramatically.

One challenge faced by individuals with low vision might be related to their general inability to generate an accurate belief vector within an environment. Normally sighted subjects can choose from rich array of landmarks to provide important information about their state within the environment; however, individuals with low vision have limited access to these perceptual landmarks. These results suggest that one of the challenges that someone with low vision might have is being able to consider multiple places simultaneously when they become disoriented within an environment.

**Conclusions**

Spatial navigation is an important cognitive process that involves a number of cognitive sub-systems. The goal of understanding spatial navigation performance is comprehending what information is made explicit in a cognitive map and how those representations are used for spatial navigation performance. The current studies suggest that under conditions of state uncertainty one of the primary factors limiting human navigation performance is spatial updating and not perception, memory access or the subject’s decision strategy.
References


